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ABSTRACT

This document describes a mathematics course that uses the computer to solve mathematics problems. It was developed to be used with students who have completed at least one year of general mathematics or are not achieving success in the traditional mathematics program. The course is intended to review, reinforce, and extend concepts included in the secondary school mathematics curricula. The major focus of the course is on problem solving. The teaching of computer programming is intended to provide a model of a problem solving process in which a problem is broken down into its component parts. The book contains 60 lessons, some of which are laboratory sessions, which use the computer and the process of computer programming to help students understand: (1) algorithms; (2) perimeters of polygons; (3) area; (4) the order of arithmetic operations; (5) number systems; (6) consumer mathematics; (7) measurement; (8) algebraic equations; (9) simple interest; and (10) volume. (TW)

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FOREWORD

Computer Mathematics: An Introduction Part I is a mathematics course that uses the computer to solve mathematics problems. Students are given opportunities to explore various branches of mathematics. They are then taught how to develop computer solutions to problems and extensions of problems in the different areas of mathematics.

These computer lessons are designed not only to make the student computer literate but also to:

a. increase his/her motivation to study mathematics,
b. improve his/her ability to conceptualize,
c. improve his/her problem solving skills,

all leading to an improvement in performance in mathematics and in other subject areas.

We believe that by implementing this course concomitant benefits will be:

a. increasing the school's holding power and
b. providing another option to meet the New York State Regents Action Plan requirements for a second year of mathematics credit for a local diploma.

This course has been approved by the New York State Education Department's Bureau of Mathematics for a second year of credit in mathematics for a local diploma.

Charlotte Frank

Charlotte Frank
Executive Director
Division of Curriculum and Instruction

5

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Kent Beaty designed the cover.

INTRODUCTION

Computer Mathematics is a course designed to engage students in using the computer to solve mathematical problems. It has been prepared to be used with students who have completed a year of general mathematics or for those who are not meeting success in the more traditional mathematics program. The purpose of this course is to review, reinforce and extend concepts from the secondary school mathematics curricula. We have found that by directing students' interest in computers toward the study of mathematics, they not only become computer literate but also better mathematics students.

The focus of this course is problem solving. The process of problem solving, as treated in this curriculum, can be compared to that which a tutor must undergo in teaching someone else. In explaining how a problem is solved, the tutor's thought processes become crystalized; thus, he/she develops a deeper understanding of how to solve a problem. When we program a computer, we must first break the problem down into its component parts which we then translate into a computer language. In this course, students are required to participate in a similar process. This activity evolves genuine problem solving skills, applicable to many situations.

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Lesson: 1

AIM: What is the process of computer programming?

PERFORMANCE OBJECTIVES: The student will be able to...

1. write the sequence of steps used in computer programming;
2. describe the role of a systems analyst, computer programmer, keypunch operator, terminal operator and computer user.

VOCABULARY: simulation, computer scientist, systems analyst, computer programmer, keypunch operator, terminal operator, user

MOTIVATION:

1. Say to the students, "I am going to give you a sequence of instructions to memorize. Each time I point to you and say, 'RUN', you will follow the instructions."
2. Instructions:
   a) Stand up.
   b) Walk to the front of the room.
   c) Write your first name on the chalkboard.
   d) Say, "I love computers."
   e) Return to your seat.

   (Make sure that the instructions are clear; you may wish to repeat them.)
3. Do the above activity 4 or 5 times.

DEVELOPMENT:

1. Tell the students that they have just simulated a computer.
   (Definition: To "simulate" is to artificially create the conditions found in real operation.)
2. Our exercise had four parts. Using computers involves four stages.
   Let us compare:

OUR EXERCISE	USING COMPUTERS
a) We had a PLAN. (The plan was to do an activity that shows what happens in computer programming.)	a) Write a PLAN. (The plan is an outline of the steps the computer must follow to solve a problem.)
b) You were given INSTRUCTIONS in English (a language you understand).	b) You give a computer INSTRUCTIONS in a language it understands. We will use BASIC (Beginner's All-Purpose Symbolic Instruction Code).
c) You MEMORIZED the instructions. (You stored them in your memory.)	c) The instructions are stored in the computer's MEMORY.
d) When you heard the word RUN, you "executed" (followed) the instructions.	d) A computer executes the instructions in its memory when it receives the proper signal.

APPLICATIONS:

Computer programming is divided into these parts:

1. The person who develops a plan for the computer to solve a problem is the COMPUTER SCIENTIST or SYSTEMS ANALYST.
2. The plan is translated into a language the computer can understand. The sequence of instructions written in a computer language is called the PROGRAM. The person who writes the program is called the COMPUTER PROGRAMMER or PROGRAMMER.
3. The program can be a) typed in or b) entered using computer cards. The person who types in the program is called the TERMINAL OPERATOR. The one who makes up the cards is called the KEYPUNCH OPERATOR.
4. The person who runs the program is called the USER.

Ask the students to examine and discuss newspaper clippings indicating the job description for each person involved in the process of computer programming.

SUMMARY:

1. What are the jobs associated with computer programming?
2. Which job(s) do you think we will be doing this term? (all of them)

HOMEWORK:

1. Describe the role of each in computer programming:
    - a) systems analyst
    - b) computer programmer
    - c) terminal operator
    - d) keypunch operator
2. Which of these jobs requires the MOST education? ...the LEAST?
3. What are the four stages of computer programming?

HOMEWORK SOLUTIONS:

1.
    - a) systems analyst - develops the plan for the computer to solve a problem
    - b) computer programmer - writes the set of instructions for the computer to solve a problem
    - c) terminal operator - types in and runs the program
    - d) keypunch operator - "punches" cards with the set of instructions
2. MOST - systems analyst
   LEAST - terminal operator
3.
    - a) Write a PLAN.
    - b) Translate the plan into instructions written in a computer language; i.e., write a PROGRAM.
    - c) Type in the program and have the computer memorize the instructions.
    - d) Execute the instructions; i.e., RUN the program.

Lesson: 2

AIM: How do we develop an algorithm?

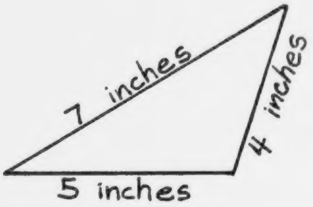
PERFORMANCE OBJECTIVES: The student will be able to...

1. find the perimeter of a triangle;
2. define "equilateral triangle";
3. write an algorithm for finding the perimeter of an equilateral triangle.

VOCABULARY: perimeter, equilateral triangle, algorithm, input, output

MOTIVATION:

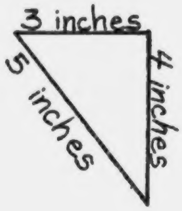
Find the perimeter of the following triangle.


7 inches
4 inches
5 inches

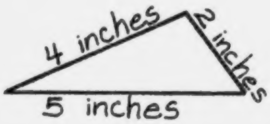
DEVELOPMENT:

1. Ask, "What do we mean by perimeter?" Ask the students to recall the definition. Perimeter is the distance around the boundary of a figure.
2. Elicit a response to the motivation problem. (16 inches) Also, ask the students to find the perimeter of each triangle below.

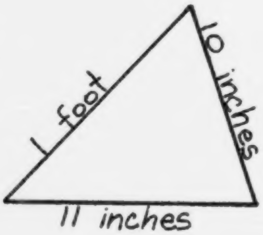
a)


3 inches
5 inches
4 inches

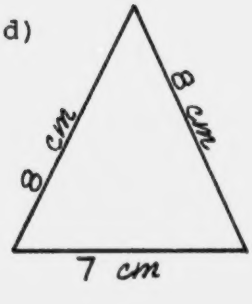
b)


4 inches
2 inches
5 inches

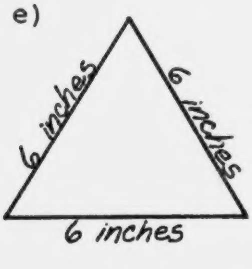
c)


1 foot
10 inches
11 inches

d)



e)


6 inches
6 inches
6 inches

3. In the last example (e), all three sides have the same length. Because of this, the triangle is considered special. Ask, "What name is given to a triangle whose sides have the same length?" Definition: An equilateral triangle is a triangle all of whose sides have the same length.

4. Find the perimeter of an equilateral triangle if each side has length a) 16 cm, b) $1\frac{1}{2}$ ft., c) 7.1 yards.

5. Say to the students that we are now ready to prepare our first computer solution for a problem. Present the problem that follows for a computer solution. Problem: Find the perimeter of an equilateral triangle if we are given the length of a side.

6. In order to use a computer to solve this, we must break the problem down into steps for the computer to follow. Definition: A step-by-step procedure that leads to the solution of a problem is called an ALGORITHM. An algorithm is a really important part of a "recipe" for getting a solution.

7. To get the algorithm for our problem, ask, "How would you find the perimeter of an equilateral triangle?" Elicit the responses, add the lengths of the three sides or multiply the length of one side by three.

6. Write the following on the chalkboard. The algorithm is: Given the length of the side (s),

   P = 3X Side or

   P = 3*S

APPLICATIONS:

1. Use P = 3xS to find the perimeter of an equilateral triangle with sides that measure

   a) 5 in.
   b) 7 mm
   c) 6.1 in.
   d) 6.3 feet
   e) 5.21 in.

   (Be careful with the decimals. Check the results by addition.)

16

2. The number we start with is often called the INPUT. The number we end with is often called the OUTPUT. Find the output if we use the algorithm P = 3xS with the following input.

   a) 16 in. b) 3.5 feet c) $1\frac{1}{4}$ in.

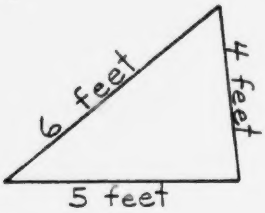
SUMMARY:

1. What is meant by the word "algorithm"?
2. Why are algorithms needed when we use computers?

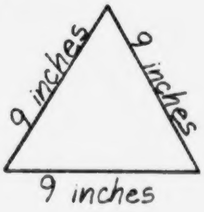
HOMEWORK:

1. Find the perimeter of each triangle.

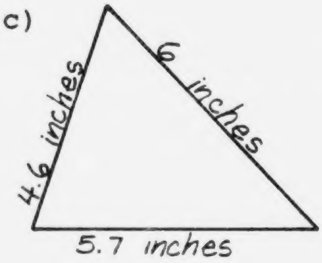
a)


6 feet
4 feet
5 feet

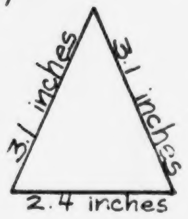
b)


9 inches
9 inches
9 inches

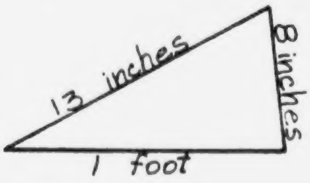
c)


4.6 inches
6 inches
5.7 inches

d)


3.1 inches
3.1 inches
2.4 inches

e)


13 inches
8 inches
1 foot

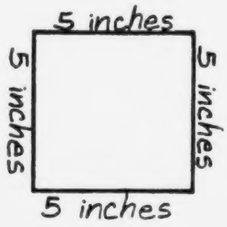
2. Use P = 3xS to find the perimeter of an equilateral triangle whose side has length:

   a) 6.2 cm
   b) 30 feet
   c) 1 mile
   d) 1.32 inches

3. Predict the output if we use P = 3xS and the input:

   a) 26 inches
   b) $7\frac{1}{2}$ in.
   c) 8.2 mm
   d) $5\frac{1}{3}$ ft.

4. Find the perimeter of this square.


5 inches
5 inches
5 inches
5 inches

5. Write an algorithm for finding the perimeter of a square.

HOMEWORK SOLUTIONS:

1. a) 15 ft.
   b) 27 in.
   c) 16.3 in.
   d) 8.6 in.
   e) 33 in.

2. a) 18.6 cm
   b) 90 ft.
   c) 3 miles
   d) 3.96 in.

3. a) 78 in.
   b) $22\frac{1}{2}$ in.
   c) 24.6 mm
   d) 16 ft. (or 192 in.)

4. 20 in.

5. P = 4xS

Lesson: 3L

AIM: How do we use an algorithm to solve a problem?

PERFORMANCE OBJECTIVES: The student will be able to...

1. incorporate an algorithm into a plan for a computer solution to a problem;
2. identify legal variables (in BASIC);
3. translate phrases from a plan into BASIC instructions;
4. describe the INPUT, PRINT and END statements.

VOCABULARY: INPUT, exponentiation, PRINT, END

MOTIVATION:

Write a program that will solve the first problem presented for a computer solution.

Problem: Find the perimeter of an equilateral triangle if we are given the length of a side.

DEVELOPMENT:

1. Say to the students that they need a PLAN in order to solve the problem. Yesterday's work will be helpful. To develop a plan, ask the students to write the answer to the following question: "How did we use P=3xS to solve the problem?"

   PLAN FOR A COMPUTER SOLUTION

   1. Put in a value for the length of a side, S.
   2. Calculate the perimeter: P = 3xS.
   3. Print the value of P (so that we know the answer).
   4. This ends the instructions.

2. Each of these steps has a corresponding "BASIC" instruction. As you write the plan on the chalkboard, be sure to leave room to the right of it.

PLAN	BASIC INSTRUCTIONS (PROGRAM)
1. Put in a value for the length of a side, S.	10 INPUT S
2. Calculate the perimeter: P = 3xS.	20 P=3*S
3. Print the value of P.	30 PRINT P
4. This ends the instructions.	40 END

3. Ask the students to
   a) enter each line of the program into the computer and
   b) run the program using the values for S given in yesterday's lesson. It is important for the students to distinguish between giving the computer the instructions (the program) and giving it the data (running the program).
4. Discuss the following ideas with the students.
   a) When the computer reaches an "INPUT" statement, a question mark will appear on the screen. This tells the user that the computer is waiting for data.
   b) For the names of memory cells (variables) we can use either
      i) one letter or
      ii) a letter followed by either another letter or a digit.
      Examples: A, B, Z, A1, S9, XY, AB
   c) Calculations in BASIC.
      i) P = 3*S means memory cell P is "assigned the value" 3 times the value of S or P "becomes" 3 times the value of S or memory cell P "holds" 3 times the value of S.
      ii) Ask the students to note the following list of symbols used for the arithmetic operations in BASIC.

Operation	BASIC Symbol
Addition	+
Subtraction	-
Multiplication	*
Division	/
Exponentiation (A^B)	A↑B or A^B on some machines

APPLICATIONS:

1. Rewrite each of the following in BASIC symbols.
   a) $2 \times 5 + 2^3$
   b) $X = 3A + AB - C \div D$
   c) $Y = 6X + W \div Z$
   d) $B = C^3$

SUMMARY:

1. What is a memory cell?

2. What variables can be used as memory cell names?

3. How is AB interpreted in BASIC?....in mathematics?

4. Explain what each instruction does.
   a) INPUT
   b) PRINT
   c) END

HOMEWORK:

1. Use P = 3xS to find the perimeter if the length of a side is:
   a) 11 in.
   b) 10.2 ft.

2. Which of the following are legal names for memory cells in BASIC?
   a) X
   b) 3X
   c) X3
   d) Z
   e) A,B
   f) Z!
   g) AB
   h) BA
   i) B*

3. Translate each of the following into BASIC.
   a) $P = 4S$
   b) $A = X^3$
   c) $Y = A \div B + 2AB$
   d) $M = NP - X \div Y$

4. What do we call the person who develops a plan for a computer solution to a problem?

HOMEWORK SOLUTIONS:

1. a) 33 in. b) 30.6 ft.

2. legal variables in a,c,d,g,h

3. a) P = 4*S c) Y = A/B + 2*A*B
   b) A = X↑3 d) M = N*P - X/Y

4. systems analyst or computer scientist

Lesson: 4

AIM: How does a computer execute one or two lines of a program?

PERFORMANCE OBJECTIVES: The student will be able to...

1. trace a (simple) program;
2. predict the result of executing one or two lines of a program.

VOCABULARY: "trace" a program, "execute" instructions

MOTIVATION:

How can we tell if a program will work (solves the problem) before running it on a computer?

DEVELOPMENT:

1. In order to check our work before running a program, we use a procedure called "tracing".

Procedure for Tracing a Program

a) In addition to showing what will appear on the screen, place a chart of the computer's memory next to your program.

b) Choose "test data" to use with the program.

c) Box off the values that will be displayed, as your follow the instructions.

d) Show all calculations. The answers should agree with what you expected.

2. Ask, "What is the perimeter of an equilateral trialgle whose sides measure 5?" Let us see if our program gives the answer 15. Trace the first program with the students as follows.

PROGRAM	MEMORY CELLS	
	S	P
10 INPUT S	5	
20 P = 3*S		3*5 =15
30 PRINT P		
40 END		
RUN		
?5		

Say to the students that since 15 is the predicted value for the perimeter, the program appears to solve the problem.

3. Ask the students to trace the same program with S = 12.
4. a) The following is a program for doubling a number. Ask the students to trace the program using the data:
   - i) 6
   - ii) .5
   - iii) 7.1
   - iv) 0

   Program

```
10  INPUT X
20  N = 2*X
30  PRINT N
40 END
```

   b) As you trace the program with the students, stress the importance of showing what will occur <u>both</u> on the screen and in the computer's memory.

   c) Elicit the trace that follows.

Screen	Memory	
	X	Y
RUN		
?6	~~6~~	
12		2*6= ~~12~~
RUN		
?.5		
1	~~.5~~	2*.5=~~1~~
RUN		
?7.1	~~7.1~~	
14.2		2*7.1= ~~14.2~~
RUN		
?O	0	
0		2*0=0

APPLICATIONS:

1. In each case, find the value the computer would store in memory cell A after the computer executes the given instruction. Use X = 2, Y = 5, Z = 10.
   - a) 70 A = X*Y
   - b) 20 A = Y - X
   - c) 90 A = Y↑X
   - d) 40 A = Z/Y
   - e) 30 A = Z/X
   - f) 80 A = X*Y*Z

2. Find the final value assigned to A if B = 3, C = 9 and each of the following pairs of instructions is executed.

    a) 30 X = B + C   b) 50 D = B*C
    b) 40 A = X*B   60 A = D + 3

3. The following program is for finding one-half of a number.

    a) Trace the program using the data 10, 50 and 11.
    b) Does the program work correctly? If not, correct it.

```
10 INPUT X
20 Y = X/2
30 PRINT X
40 END
```

(Note: In this program the number entered is being displayed. Ask, "In what memory cell is the value for one-half of the number entered stored?" Change line 30 to PRINT Y.)

SUMMARY:

What is the purpose of tracing a program?

HOMEWORK:

1. Translate into BASIC.

    a) $X = 4_y$ c) A = B x C + D
    b) $Y = X^2$ d) A = B÷3xC

2. Use the algorithm P = 3*S to find P if S =

    a) 12   b) 1.6

3. If A = 6 and B = 3, find the value assigned to C in each of the following.

    a) 100 C = A - B   c) 90 C = A↑B
    b) 60 C = A*B   d) 160 C = 5*A/B

4. In each of the following, find the value stored in memory cell Z after the computer executes each pair of instructions in order. Use X = 10 and Y = 2.

    a) 10 A = X/Y   c) 120 W = 10↑Y
        20 Z = A + 5   130 Z = W/Y
    b) 70 B = X - Y
        80 Z = B*2

HOMEWORK SOLUTIONS:

1. a)	X = 4*Y	c)	A = B*C + D
b)	Y = X↑2	d)	A = B/3*C
2. a)	36	b)	4.8
3. a)	3	c)	216
b)	18	d)	10
4. a)	10	c)	50
b)	16		

Lesson: 5L

AIM: How do we predict the output of a program?

PERFORMANCE OBJECTIVES: The student will be able to...

1. successfully trace a simple program;
2. predict the output of a program;
3. run a program after it has been traced.

VOCABULARY: trace, execute, output

MOTIVATION:

Write the program on the chalkboard and ask the students to answer the questions.

Program

```
10 INPUT X,Y
20 Z = X*Y
30 PRINT X
40 PRINT Y
50 PRINT Z
60 END
```

1. What will appear on the screen if the command RUN is typed into the computer?
2. If X = 4 and Y = 7, what will the output look like?

DEVELOPMENT:

1. Elicit the following solutions to the motivation problems.

   For 1,

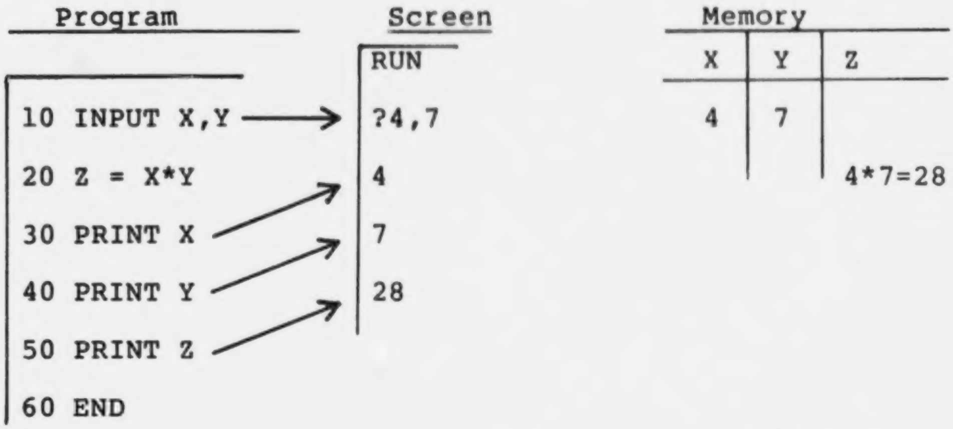
   ```
   RUN
   ?
   ```

   appears on the screen.

   Because of the INPUT statement in line 10, a question mark will appear on the screen.

   For 2, we must trace the program to predict the output.

2. a) Write on the chalkboard:



Program

```
10 INPUT X,Y
20 Z = X*Y
30 PRINT X
40 PRINT Y
50 PRINT Z
60 END
```

Screen

```
RUN
?4,7
4
7
28
```

Memory

X	Y	Z
4	7	
		4*7=28

b) Ask, "How does the computer 'know' that <u>two</u> numbers are being entered?" Elicit that a comma is used to separate the numbers.

c) Ask, "What appears on the screen when line 20 is executed?" Point out that the results in line 20 are <u>internal</u> only. They are <u>not</u> shown on the screen.

3. Ask the students to enter the program into the computer and run it. Ask them to note that
   a) the RETURN (or ENTER) key must be pressed when a line of the program or data has been entered and that
   b) typing out LIST and pressing the RETURN (or ENTER) key will show the instructions on the screen.

APPLICATIONS:

For each, trace the program using the given data, predict the output, and then verify your prediction by running it on the computer.

1.

```
10 INPUT X,Y
20 Z = X + Y
30 PRINT X
40 PRINT Y
50 PRINT Z
60 END
```

a) Use X=8, Y=4.

b) Use X=.6, Y=.9.

2.

```
10 INPUT A,B,C
20 D = 2*A
30 E = B/C
40 PRINT A
50 PRINT B
60 PRINT C
70 PRINT D
80 PRINT E
90 END
```

a) Use A=5, B=10, C=2.

b) Use A=0, B=6.4, C=2.

3.

```
10 INPUT A,B
20 PRINT A
30 C = A/B
40 PRINT B
50 D = 5*C
60 PRINT C
70 PRINT D
80 END
```

a) Use A=12, B=3.

b) Use A=4, B=2.

SUMMARY:

Describe the procedure for tracing a program.

HOMEWORK:

1. If A = 2, B = 6 and C = 4, predict the result of executing each instruction.
   - a) `10 D = B*C*A`
   - b) `90 D = B↑A`
   - c) `100 D = A↑B`

2. Predict the value stored in memory cell E as a result of executing the two instructions in each if A = 10, B = 20, C = 5.

a)

```
10 D = A*B
20 E = D/C
```

b)

```
30 D = B + C - A
40 E = D*2
```

c)

```
50 D = A + B
60 E = D + C
```

3. For each, trace the program using the given data.

a)

```
10 INPUT M,N
20 P = M - N
30 PRINT M
40 PRINT N
50 PRINT P
60 END
```

Use M=5, Z=2.

b)

```
10 INPUT C,X
20 Y = X↑3
30 Z = C*Y
40 PRINT Y
50 PRINT Z
60 END
```

Use C=10, X=2.

HOMEWORK SOLUTIONS:

1. a) 48
   b) 36
   c) 64

2. a) 40
   b) 30
   c) 35

3. a) Screen

```
RUN
?5,2
5
2
3
```

Memory

M	N	P
5	2	
		5-2=3

b) Screen

```
RUN
?10,2
8
80
```

Memory

C	X	Y	Z
10	2		
		$2^3=8$	
			10*8=80

Lesson: 6

AIM: How do we develop a complete computer solution for determining the perimeter of a regular polygon?

PERFORMANCE OBJECTIVES: The student will be able to...

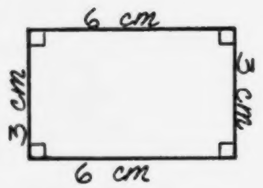
1. define "regular polygon";
2. find the perimeter of a regular polygon, given the length of a side;
3. develop a complete computer solution for determining the perimeter of a regular polygon.

VOCABULARY: regular polygon, perimeter, rhombus, rectangle, square, equilateral, equiangular, quadrilateral

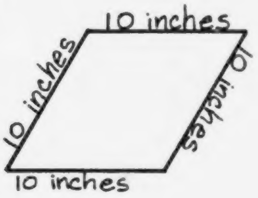
MOTIVATION:

1. In each case find the perimeter.

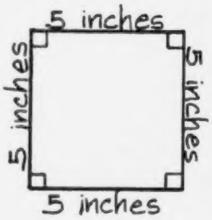
a)



b)



c)



2. How are the figures the same? What makes the figures different?

DEVELOPMENT:

1. Elicit responses to the motivation problems. Ask the students to notice that all the figures have four sides. Say that four-sided polygons are called quadrilaterals. In figure a), the angles all have the same measure. Such a polygon is called equiangular. An equiangular quadrilateral is a rectangle.
In figure b), the sides all have the same measure. Such a polygon is called equilateral. (Say to the students

that "lateral" means side.) An equilateral quadrilateral is a rhombus.

In figure c), the polygon is both equilateral and equiangular at the same time. Such a polygon is called a regular polygon. A regular quadrilateral is a square.

2. Today we will develop a complete computer solution for problems involving finding the perimeter of regular polygons.

Problem: Find the perimeter of a regular polygon when we are given the number of sides and the length of a side.

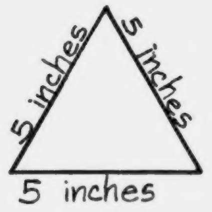
a) Ask the students to recall that there are 3 stages in finding a complete computer solution to a problem.

   i) ANALYSIS - Develop an algorithm... how would we solve the problem?
   ii) PLAN - List the steps for the computer to be able to use the algorithm to solve the problem.
   iii) PROGRAM - Translate the plan into BASIC.

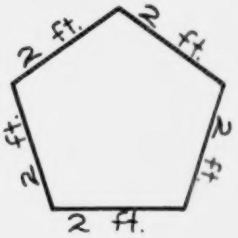
b) Elicit the following solution to the problem.

i) Analysis - Let us try a few examples to help "see" how to solve the problem.

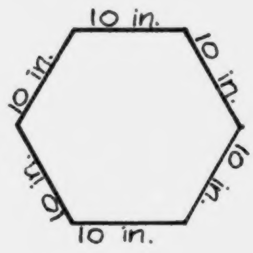
1. # of sides: 3
   length of each side: 5 in.
   P = 5 + 5 + 5 = 3(5) = 15 in.



2. # of sides: 5
   length of each side: 2 ft.
   P = 2+2+2+2+2 = 5(2) = 10 ft.



3. # of sides: 6
   length of each side: 10 in.
   P = 10+10+10+10+10+10 = 6(10) = 60 in.



Elicit that in each case, to find perimeter, we multiply the number of sides by the length of each side.

Symbolically, our algorithm becomes: Given a value for N and S, P = N x S, where P is the perimeter, N is the number of sides, and S is the length of a side.

ii) Plan -

1) Put in values for the number of sides and the length of each side: N,S.
2) Calculate the perimeter: P = NxS.
3) Print P.
4) End.

iii) Program

```
10 INPUT N,S
20 P = N*S
30 PRINT P
40 END
```

3a. Ask the students to trace the program when N=3 and S=5 as follows.

Screen

```
RUN
?3,5
15
```

Memory

N	S	P
3	5	
		3*5=15

b) If there is time and computers are available, ask the students to run the program using the data obtained earlier in the lesson.

SUMMARY:

How do you develop a computer solution to a problem?

HOMEWORK:

1. If A = 2 and B = 3, predict the result in each of executing the statement or statements.

a) `10 Y = A*B*10`

b) `10 X = B↑A`

c) `X = A↑B`

d)
```
10 X = A + B
20 Y = 2*X
```

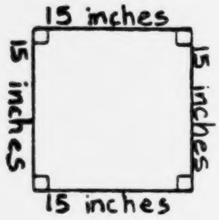
2. Trace the program using the given data.

```
10 INPUT X,Y
20 A = 2*X*Y
30 B = X↑2
40 PRINT A
50 PRINT B
60 END
```

Use X = 5, Y = 6.

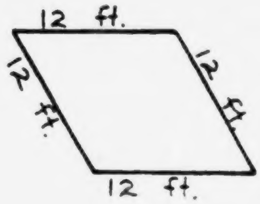
3. In each case, find the perimeter and classify the type of polygon as regular, equilateral or equiangular.

a)



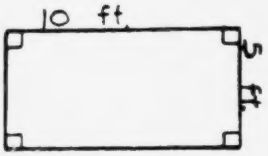
15 inches
15 inches
15 inches
15 inches

b)

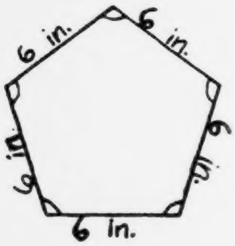


12 ft.
12 ft.
12 ft.
12 ft.

c)

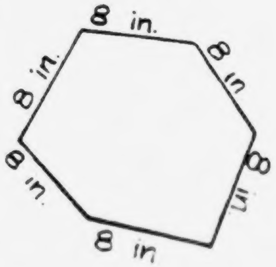


d)



6 in.
6 in.
6 in.
6 in.
6 in.

e)



8 in.
8 in
8 in
8 in
8 in
8 in.

HOMEWORK SOLUTIONS:

1. a) 60 c) 8
   b) 9 d) 10

2. Screen

```
RUN
?5,6
60
25
```

Memory

X	Y	A	B
5	6		
		2*5*6=60	
			5^2=25

3. a) P = 60 in., regular (square)
   b) P = 48 in., equilateral (rhombus)
   c) P = 30 ft., equiangular (rectangle)
   d) P = 30 in., regular
   e) P = 48 in., equilateral

Lesson: 7L

AIM: How do we develop an algorithm to find the perimeter of a rectangle?

PERFORMANCE OBJECTIVES: The student will be able to...

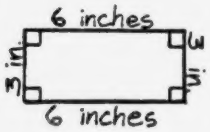
1. find the perimeter of a rectangle;
2. develop an algorithm to find the perimeter of a rectangle;
3. incorporate the algorithm into a plan to find the perimeter of a rectangle;
4. write a program in BASIC for finding the perimeter of a rectangle.

VOCABULARY: rectangle, perimeter

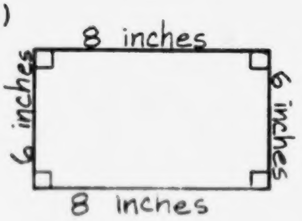
MOTIVATION:

Find the perimeter of each of the following rectangles:

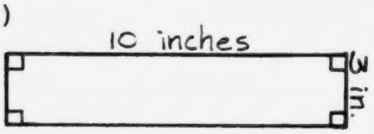
a)



b)



c)



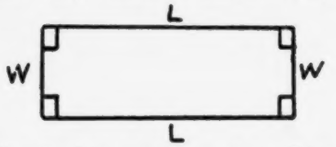
DEVELOPMENT:

1. Ask, "Can you find an algorithm for the computer to use in finding the perimeter of a rectangle?" In example c above, point out that only one length and one width is given. Ask, "Why is only one length and width needed?"
2. Ask the students to develop an algorithm as follows: Given values for the length (L) and width (W),

   $P = L + W + L + W$

   $P = L + L + W + W$

   $P = 2 \times L + 2 \times W$

   Ask the students to note that in order to develop this algorithm, we first used numbers, then generalized!)

2. Ask the students to use the algorithm with motivation examples a, b, c.

APPLICATIONS:

Present the following problem for a computer solution.

<u>Problem</u>: Find the perimeter of a rectangle when we are given the length and the width.

(Analysis was done above.)

1. a) Ask the students to develop a plan and program at their seats.
   b) The teacher should circulate to give the individual help students need. As the students develop the plan, guide them with the following questions.
      i) How do we <u>use</u> the algorithm to solve the problem?
      ii) What information do we need to enter into the computer?
      iii) What algorithm do we want the computer to use in order to determine the perimeter of each rectangle?
      iv) In what memory cell is the perimeter that we want displayed stored?
   c) Students should be sent to the computer upon finishing their work. (If the work has errors, ask them to trace their programs.)
   d) Call upon individuals to place their results on the chalkboard.

One such solution is:

<u>Plan</u>:

1) Put in values for the length and width: L, W.
2) Calculate the perimeter: P = 2*L + 2*W.
3) Print P.
4) End.

<u>Program</u>:

```
10 INPUT L,W
20 P = 2*L + 2*W
30 PRINT P
40 END
```

SUMMARY:

What is the algorithm for finding the perimeter of a rectangle? (Answers: Given a value for the length (L) and width (W), P = 2*L+2*W)

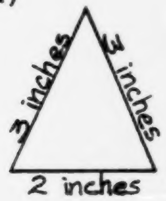
HOMEWORK:

1. Find the perimeter of each polygon.
    a) regular octagon (8 sides), each side has length 6 in.
    b) regular decagon (10 sides), each side has length 4 m
    c) equilateral dodecagon (12 sides), each side has length 5 ft.
    d) regular heptagon (7 sides), each side has length 3 in.
    e) a rectangle with length 4 in. and width 1 in.
    f) a rectangle with length 8 in. and width 7 in.
    g) a rectangle with length 15 in. and width 1 ft.
    h) a rectangle with length 7.5 in. and width 2.5 in.

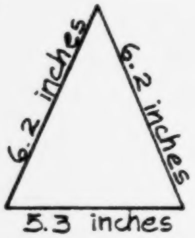
(Examples 2, 3 and 4 are important for tomorrow's lesson.)

2. An isosceles triangle is a triangle in which at least two sides have the same length. These sides are called legs. The third side is called the base. Find the perimeter of each isosceles triangle.

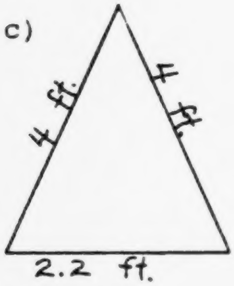
a)



b)



c)



3. Using L for the length of each leg of an isosceles triangle and B for the length of its base (the third side), write an algorithm for finding the perimeter (P) of an isosceles triangle.

4. Devise a plan and code a program for finding the perimeter of an isosceles triangle, given the lengths of its legs and base.

HOMEWORK SOLUTIONS:

1.
   - a) 48 in.
   - b) 40 m
   - c) 60 ft.
   - d) 21 in.
   - e) 10 in.
   - f) 30 in.
   - g) 54 in.
   - h) 20 in.

2.
   - a) 8 in.
   - b) 17.7 in.
   - c) 10.2 ft.

3. P = B + 2L (or P = 2L + B)

4. Plan:
   1) Put in values for the lengths of each leg and the base: L and B.
   2) Calculate the perimeter: P = B+2L (or P = 2L+B).
   3) Print the perimeter, P.
   4) End.

   Program:

```
10 INPUT L,B
20 P = B + 2*L
30 PRINT P
40 END
```

Lesson: 8

AIM: What is the order of performing the arithmetic operations?

PERFORMANCE OBJECTIVES: The student will be able to...

1. perform calculations correctly using the order of operations;
2. predict the value the computer will assign to a variable in a statement involving the order of operations.

VOCABULARY: order of operations, isosceles triangle

MOTIVATION:

Find the perimeter of an isosceles triangle in which...

a) each leg measures 5 in. and the base measures 4 in.

b) each leg measures 2 in. and the base measures $\frac{1}{2}$ in.

(Hint: Use last night's homework as a model.)

DEVELOPMENT:

1. Use the problems listed above and last night's homework to initiate a discussion of the algorithm for the perimeter of an isosceles triangle: Given values for B and L, P = B + L + L or P = B + 2L (In mathematics, we often write P = B + 2L; the multiplication is understood by us. In computer work, we must write P = B + 2*L since the computer does NOT automatically assume multiplication.)
2. Ask the students to use P = B + 2L to complete example a above (L = 5, B = 4). As you circulate around the room, you may notice two methods being used; e.g.,

METHOD 1	OR	METHOD 2
P = B + 2L		P = B + 2L
P = 5 + 2x4		P = 5 + 2x4
P = 7x4		P = 5 + 8
P = 28		P = 13

3. Ask the students to notice that method 2 yields the correct answer (show in a diagram). Performing the operations from left to right gives the wrong answer here. Tell the students that the computer performs the arithmetic operations in the same way that we do in mathematics; namely, it uses the order of operations. (Tell the students that some calculators do not.) Ask the students to review the order of operations.

4. ORDER OF PERFORMING OPERATIONS USED IN MATHEMATICS AND IN BASIC
   1) Perform all operations indicated within the parentheses.
   2) Handle any exponents that may appear.
   3) Perform all multiplications and divisions, in order, from left to right.
   4) Perform all additions and subtractions, in order, from left to right.

APPLICATIONS:

1. Find x.
   a) $x = 4 + 2 \cdot 3$
   b) $x = 10 - 2^3$
   c) $x = 7 + 5(3+9)$

2. Find the values assigned to X in each computer instruction. Use A = 12 and B = 3.
   a) `30 X = A - 2*B`
   b) `70 X = (A - 2)*B`
   c) `90 X = 2*A + 3*B`
   d) `60 X = (A+B)/3`
   e) `40 X = A + B/3`
   f) `170 X = A - 2↑3`

SUMMARY:

1. What is the order of operations in BASIC?

2. What is the purpose in having an order of operations in BASIC?

HOMEWORK:

1. Find the perimeter of each polygon.
   a) a regular icosagon (20 sides), each side has length 4 in.

b) a rectangle with length 10 in. and width 3.5 in.

c) an isosceles triangle with base 4 ft. and legs that measure 6 ft. each

2. For each of the following, find x.
   - a) $x = 6 + 3^4$
   - b) $x = 5 + 2(6-3)$
   - c) $x = 8 + 2 \cdot 7$
   - d) $x = 250 \div 10 + 40$
   - e) $x = 250 \div (10 + 40)$
   - f) $x = 90 - 3 \cdot 10 + 5^2$

3. If A = 15, B = 5 find the value of X as a result of the instruction or instructions.
   - a) `10 X = (A-B)/B`
   - b) `60 X = A - B/B`
   - c) `70 Y = A*B + 5`
     `80 X = Y + 5`
   - d) `70 X = A*B/3+2`
   - e) `90 X = A*2 - B*3`

4. The side of an octagon (8 sides) has length S. Write an algorithm for finding the perimeter.

HOMEWORK SOLUTIONS:

1. a) 80 in.
   b) 27 in.
   c) 16 ft.

2. a) 87
   b) 11
   c) 22
   d) 65
   e) 5
   f) 85

3. a) 2
   b) 14
   c) 85
   d) 27
   e) 15

4. P = 8*S

Lesson: 9L

AIM: How do we predict the output in programs involving the order of operations?

PERFORMANCE OBJECTIVES: The student will be able to...

1. predict the value the computer assigns to a variable in programs involving the order of operations;
2. make the changes in a program to obtain a desired output.

VOCABULARY: order of operations, debugging a program, editing a program, "memory map"

MOTIVATION:

The listing is a program that was just typed into a computer. Predict the output when L=5 and W=7. Include a "memory map".

```
10 INPUT L,W
20 P = 2*L + W
30 PRINT P
40 END
```

DEVELOPMENT:

1. Elicit the trace that follows.

```
Screen              Memory
RUN                  L  |  W  |  P
                    ----+-----+------
?5,7                 5  |  7  |
17                                 2*5+7 = 10+7 = 17
```

2. Ask, "Can this program be used to find the perimeter of a rectangle? (No, since only the length is doubled.) How can we adjust this program so that it will find the perimeter of a rectangle?" Elicit that we can retype line 20; e.g., either 20 P = 2*L+2*W or P = 2*(L+W). Ask the students to be sure to place an * between the numeral "2" and the left parenthesis. Tell the students that correcting an error in a program is called <u>editing</u> or <u>debugging</u> a program.

3. Ask the students to enter the original program, run it and then revise line 20. After running the program again, ask them to compare the outputs.

APPLICATIONS:

Ask the students to work on the following at their seats. Send them to the computer only after they have a) predicted the computer output and b) revised the program to get the desired output.

1)

```
10 INPUT A,B
20 C = (A+B)*3
30 PRINT A
40 PRINT B
50 PRINT C
60 END
RUN
?1,3
```

Desired Output:

```
RUN
?1,3
1
3
10
```

2)

```
10 INPUT A,B,C
20 D = A + B*C
30 PRINT D
40 PRINT C
50 PRINT B
60 PRINT A
70 END
RUN
?2,3,4
```

Desired Output:

```
RUN
?2,3,4
2
3
4
20
```

3)

```
10 INPUT X,Y,Z
20 A = X + Y*Z - Z
30 PRINT X
40 PRINT Y
50 PRINT Z
60 PRINT A
70 END
RUN
?1,2,3
```

Desired Output:

```
RUN
?1,2,3
1
2
3
6
```

(Note: All three problems involve changing the order of operations.)

After running the programs, ask the students to explain their changes.

SUMMARY:

1. What is the order of operations in computer instructions?
2. If A = 4 and B = 10, what is the value assigned to C in the computer instruction, 20 C = B-A*2?
   Change this instruction so that the value assigned to C will be 12.

HOMEWORK:

1. Find the perimeter of ...
   - a) a rectangle with length 13 in. and width 1 ft.
   - b) an equilateral triangle with each side measuring 3 in.
   - c) a pentagon (5-sided polygon) with side lengths of 6.8 in.

2. Find x.
   - a) $x = 2 + 3\cdot 4$
   - b) $x = 5 + 3(10-2)$
   - c) $x = 100 \div 4\cdot 5$
   - d) $x = 20 - 3^2 + 2^3$
   - e) $x = 20 - (3^2+2^3)$

3. If A = 10 and B = 4, find the value stored in memory cell Y as a result of performing the following instruction or instructions.

a)
```
10  Y = A - B/2
```

b)
```
30 Y = (A-B)/2
```

c)
```
90  C = A - B
100 Y = C↑2
```

d)
```
70 S = A*B
80 Y = S/2 - 5
```

4.
```
10 INPUT A,B
20 D = A - B*3

30 PRINT A
40 PRINT B
50 PRINT D
60 END
RUN
?10,2
```

a) Predict the output.

b) Revise the program so that it will produce this output.

```
RUN
?10,2
10
2
24
```

c) Revise this program so it will produce this output.

```
RUN
?10,2
10
2
2
```

HOMEWORK SOLUTIONS:

1. a) 50 in. b) 9 in. c) 34 in.

2. a) $x = 14$ b) $x = 29$ c) $x = 125$
   d) $x = 19$ e) $x = 3$

3. a) Y = 8 b) Y = 3

c) Y = 36 d) Y = 15

4. Screen

a)

```
RUN
?10,2
2
4
```

Memory

A	B	D
10	2	10 - 2*3
		10 - 6 = 4

b) 20 D = (A-B)*3

c) 20 D = A - B 3 or

20 D = A - B*4

Lesson: 10L

AIM: How do we get computers to print messages?

PERFORMANCE OBJECTIVES: The student will be able to...

1. write instructions to have the computer print messages;
2. write instructions using commas with PRINT statements;
3. write instructions using semicolons with PRINT statements.

VOCABULARY: user-friendly

MOTIVATION:

The following output appears on a computer screen.

```
RUN
?5
15
```

What problem does the program that produces it solve?

DEVELOPMENT:

1. Elicit from the students that this output does not provide enough information to answer the motivation question. It may be produced by our first program where the algorithm P=3S was used; however, from the information given on the screen we do not know that for sure.
2. Say to the students that when we write a program, one of our aims should be to make the output easy for the user to understand. We say that such a program is <u>user-friendly</u>.
3. a) Ask the students to examine the program that follows.

```
10 PRINT "HELLO"
20 INPUT X
30 PRINT "I CAN TELL YOU X"
40 PRINT "I CAN TELL YOU";X
50 END
```

b) Point out to the students that whatever appears between the quotes will be printed as a message.

c) Ask, "What is the difference between the output of lines 30 and that of line 40?" Ask the students to run the program using 10 for X.

```
RUN
HELLO
?10
I CAN TELL YOU X
I CAN TELL YOU 10
```

Elicit that when the letter X is inside the quotes, it is displayed but when it is outside the quotes the number stored in memory cell X is displayed.

4. Ask the students to compare the following two programs.

a)

```
10 INPUT X,Y,Z
20 PRINT X;Y;Z
30 END
```

b)

```
10 INPUT X,Y,Z
20 PRINT X,Y,Z
30 END
```

Say to the students that when we use semicolons with a PRINT statement, the values are printed next to each other. When we use commas with a PRINT statement, the values are printed apart as in columns. The number of spaces is dependent upon the computer being used.

APPLICATIONS:

Show the computer output for each.

1.

```
10 INPUT X
20 PRINT "X=";X
30 PRINT "GOODBYE"
40 END
(Use X = 5.)
```

2.

```
10 INPUT A,B
20 INPUT "A","B"
30 END
(Use A = 4, B = 2.)
```

3.

```
10 INPUT A,B
20 PRINT A,B
30 END
(Use A = 4, B = 2.)
```

4.

```
10 INPUT F,S
20 PRINT F+S;F-S
30 END
(Use F = 10, S = 2.)
```

5.

```
10 INPUT A,B,C
20 PRINT "A=";A
30 PRINT "B=";B
40 PRINT C
50 END
(Use A = 2, B= 3, C = 10.)
```

SUMMARY:

Describe the result of using each with a PRINT statement.

a) quotation marks
b) a comma
c) a semicolon

HOMEWORK:

For each program listed in 1-5, show the complete output.

1.
```
10 INPUT X,Y
20 Z=(X+2)*Y+2
30 PRINT Z
40 END
RUN
?5,3
```

2.
```
10 INPUT A,B
20 C = A + B/2
30 PRINT A,B,C
40 END
RUN
?3,10
```

3.
```
10 PRINT "TELL ME A NUMBER"
20 INPUT N
30 PRINT "YOUR NUMBER IS";N
40 END
END
RUN
?15
```

4.
```
10 X = 18
20 Y = 2
30 z = X/Y*3
40 PRINT X,Y,Z
50 END
```

5.
```
10 X = 10
20 Y = 3
30 Z = 4
40 A = X*(Z↑Y)
50 PRINT " X=";X;"Y=";Y;"Z=";Z
60 PRINT "A=";A
70 END
RUN
```

6. Rewrite the program for finding the perimeter of an equilateral triangle so that the output has the title, EQUILATERAL TRIANGLE PROGRAM.

HOMEWORK SOLUTIONS:

(Note: The students should include a "memory map" with each program.)

1.
```
RUN
?5,3
23
```

2.
```
RUN
?3,10
3     10     8
```

3.
```
RUN
TELL ME A NUMBER
?15
YOUR NUMBER IS 15
```

4.
```
RUN
18      2       27
```

5.
```
RUN
X=10 Y=3 Z=4
A = 640
```

6.
```
10 PRINT "EQUILATERAL TRIANGLE PROGRAM"
20 INPUT S
30 P = 3*S
40 PRINT P
50 END
```

Lesson: 11

AIM: How do we edit a program?

PERFORMANCE OBJECTIVES: The student will be able to...

revise a program to produce a given planned output, by using the "correct" PRINT statement.

VOCABULARY: edit, planned output

MOTIVATION:

1. What is the output for this program?

```
10 INPUT A
20 PRINT A
30 END
RUN
?8
```

2. Change the program so that it will produce this output.

```
RUN
?8
A=8
```

DEVELOPMENT:

1. Elicit responses to the motivation questions. For question 2, change line 20 to 20 PRINT "A=";A. Discuss with the students the use of 20 PRINT "A="; A vs. 20 PRINT "A=8".
2. Say to the students, "In the second example, I gave you the output I wanted produced. This is called a planned output."
   Definition: A PLANNED OUTPUT is the output you wish the program to produce.

3a. In the example that follows, you will be given a planned output and a program. The program will not produce this output. Revise (edit) the program so it does produce the required output.

Program

```
10 INPUT N
20 PRINT "YOUR NUMBER IS",N
30 PRINT "GOODBYE"
40 END
```

Planned Output

```
RUN
NUMBER PROGRAM
?10
YOUR NUMBER IS 10
GOODBYE
```

3b. Elicit the changes that are listed below.

```
5  PRINT "NUMBER PROGRAM"
20 PRINT "YOUR NUMBER IS";N
```

Ask, "Why must the line number be less than 10 for the first change?"

On some computers, blanks are automatically inserted before and/or after a displayed number. Check your manual and computer and adjust the program accordingly.

APPLICATIONS:

For each program and planned output, edit the program so that it will produce the planned output. (If there is time and they are in the laboratory, the students can be sent to the computers to verify their work.)

1. Planned Output

```
RUN
JOHN'S PROGRAM
?5,6
A=5
B=6
RUN
?3,2
A=3
B=2
```

Program

```
10 "JOHN'S PROGRAM"
20 INPUT X,Y
30 PRINT "A"
40 PRINT B=
50 END
```

2. Planned Output

```
RUN
TELL ME THE LENGTH
?6
PERIMETER IS 18
RUN
TELL ME THE LENGTH
?10
PERIMETER IS 30
```

Program

```
10 INPUT 6
20 P=3*S
30 PRINT P
40 END
```

3. Planned Output

```
RUN
ENTER A NUMBER
?50
YOUR NUMBER IS 50
DOUBLE IS 100
RUN
ENTER A NUMBER
?7.5
YOUR NUMBER IS 7.5
DOUBLE IS 15
```

Program

```
10 ENTER A NUMBER
20 INPUT N
30 PRINT N
40 X=2N
50 PRINT X
60 END
```

SUMMARY:

How do we get the computer to print a message?

HOMEWORK:

1. Find the perimeter of an isosceles triangle with a leg of length 6.5" and a base of length 4".
2. Show the complete output.

```
10 PRINT "NUMBER PROGRAM"
20 INPUT X,Y
30 PRINT "X=";X;"Y=";Y
40 Z = X+Y
50 PRINT "SUM OF X AND Y =";Z
60 END
RUN
NUMBER PROGRAM
?6,11
```

Follow the instructions for questions 3 and 4.

a) Show the output for the old program.

b) Edit the old program so that it will produce the new output, keeping as much of the old program as possible.

3. Old Program

```
10 INPUT X
20 Y = X+10
30 PRINT X
40 END
RUN
?20
```

New Output

```
ENTER A NUMBER
?20
ADD 10 TO GET 30
```

4. Old Program

```
10 INPUT A
20 Y = A 2
30 PRINT Y
40 END
RUN
5
```

New Output

```
SQUARING
TELL ME A NUMBER
?5
THIS SQUARED IS 25
```

HOMEWORK SOLUTIONS:

1. 17"

(Note: Ask the students to include a "memory map" in 2-4.)

2.

```
RUN
NUMBER PROGRAM
?6,11
X=6 Y=11
SUM OF X AND Y = 17
```

3. a)

```
RUN
?20
30
```

b)

```
5  PRINT "ENTER A NUMBER"
10 INPUT X
20 Y = X+10
30 PRINT "ADD 10 TO GET";Y
40 END
```

4. a)

```
RUN
?5
25
```

b)

```
5  PRINT "SQUARING"
7  PRINT "TELL ME A NUMBER"
10 INPUT A
20 Y = A 2
30 PRINT "THIS SQUARED IS";Y
40 END
```

Note: Students may NOT number the programs as above. This can be discussed in going over the homework AFTER the work in lesson 12L is gone over.

Lesson: 12L

AIM: What do we know about editing a program?

PERFORMANCE OBJECTIVES: The student will be able to...

1. edit a program using the correct PRINT statements;
2. revise a program leaving the original line numbers unchanged.

VOCABULARY: edit

MOTIVATION:

Revise our very first program to make it a little more user-friendly.

Old Program

```
10 INPUT S
20 P = 3*S
30 PRINT P
40 END
```

Planned (New) Output

```
RUN
HELLO! - I WILL FIND THE
PERIMETER OF YOUR EQUILATERAL TRIANGLE.
***************************************
TELL ME THE LENGTH OF A SIDE.
?12
THE PERIMETER IS 36
GOODBYE!
```

DEVELOPMENT:

1. Ask the students to enter the old program into the computer and run it.
2. Develop with the students the program that follows. Notice that the program does not have line numbers.

```
PRINT "HELLO! - I WILL FIND THE"
PRINT "PERIMETER OF YOUR EQUILATERAL TRIANGLE."
PRINT "***************************************"
PRINT "TELL ME THE LENGTH OF A SIDE."
INPUT S
P = 3*S
PRINT "THE PERIMETER IS";P
PRINT "GOODBYE!"
END
```

3. Discuss the line numbers.
   - a) Say that we can number 10, 20, ...90; but then, we would have to retype the entire program.
   - b) Say that it would be preferable to keep the original program and make as few changes as possible. We do this by
     - i) determining which instructions are the same and
     - ii) numbering the new instructions around these. Then we need to type in only the new instructions.

```
Original Program         New Instructions
  10 INPUT S          2  PRINT "HELLO! - I WILL FIND THE"
  20 P = 3*S          3  PRINT "PERIMETER OF YOUR EQUILATERAL TRIANGLE."
  30 PRINT P          5  PRINT "***************************************"
  40 END              7  PRINT "TELL ME THE LENGTH OF A SIDE."
                      30 PRINT "THE PERIMETER IS";P
                      35 PRINT "GOODBYE!"
```

Ask the students to note that the new line 30 will automatically erase the previous one when we type it in and press RETURN (or ENTER). (Note: An overhead projector will simplify the illustration of the line numbering scheme above.)

4. Ask the students to enter the new instructions, type in LIST and press RETURN (or ENTER) and then RUN the revised program.

5. Go over the homework from last night, applying these new ideas.

APPLICATIONS:

First show the output of the old program. Edit it so that it will produce the new output. Keep as much of the old program as possible. (Ask the students to RUN the new programs as soon as they have written them.)

```
1. Old Program                 New Output
   10 INPUT N,D                ENTER TWO NUMBERS
   20 Q = N/D                  ?10,5
   30 PRINT Q                  THE QUOTIENT IS 2
```

```
    40 END
```

2. Old Program

```
10 INPUT M,TU,W,TH,F
20 S = M+TU+W+TH+F
30 PRINT S
40 END
```

New Output

```
MONEY EARNED THIS WEEK
ENTER WHAT YOU EARNED FOR FIVE DAYS.
?40.20,93.65,75,86,100.38
YOU EARNED $395.23
```

SUMMARY:

How do we edit a program <u>without</u> changing any of the lines from the original program?

HOMEWORK:

Evaluate.

1. 2 + 3 x 6 - 5
2. 30 - 10 ÷ 2
3. (4 + 3) x 8
4. 3 x 2 ÷ 6

Show the complete output.

5.

```
10 X=2
20 Y=5
30 PRINT "ANSWERS"
40 PRINT "X=",X
50 PRINT "Y=",X
60 END
RUN
```

6.

```
10 INPUT X,Y
20 Z = X↑2 + Y↑2
30 PRINT "Z=";Z
40 PRINT "I LOVE COMPUTERS!"
50 END
RUN
?5,4
```

7. a) For the old program, show the output.
   b) Edit the old program so that it will produce the new output.

Old Program

```
10 INPUT X
20 Y = X*2
30 Z = X*3
40 PRINT X
50 PRINT Y
60 PRINT Z
70 END
RUN
?8
```

New Output

```
DOUBLING AND TRIPLING YOUR NUMBER
TELL ME YOUR NUMBER.
?8
YOUR NUMBER DOUBLED IS 16
YOUR NUMBER TRIPLED IS 24
SINGLE-DOUBLE-TRIPLE-HOMERUN!
END
```

HOMEWORK SOLUTIONS:

1. 15   2. 25   3. 56   4. 1

(Note: Ask the students to include a "memory map.")

5.

```
RUN
ANSWERS
X = 2
Y = 5
```

6.

```
RUN
?5,4
Z = 41
I LOVE COMPUTERS!
```

7. a)

```
RUN
?8
8
16
24
```

b)

```
 5 PRINT "DOUBLING AND TRIPLING YOUR NUMBER"
 7 PRINT "TELL ME YOUR NUMBER."
10 INPUT X
20 Y = X*2
30 Y = X*3
40 PRINT "YOUR NUMBER DOUBLED IS";Y
50 PRINT "YOUR NUMBER TRIPLED IS";Z
60 PRINT "SINGLE-DOUBLE-TRIPLE-HOMERUN!"
65 PRINT "END"
70 END
```

Lesson: 13

AIM: How do we design a display when writing a complete computer solution to a problem?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the average of two numbers;
2. develop a complete computer solution to a problem, including a well-designed "planned output".

VOCABULARY: mean, average

MOTIVATION:

Find the average (arithmetic mean) of 70 and 80.

DEVELOPMENT:

1. Say to the students that today they will learn to develop a computer solution to a problem from start to finish. This includes
    a) developing an ALGORITHM to solve the problem;
    b) writing a PLAN for the computer to use the algorithm;
    c) designing a PLANNED OUTPUT;
    d) coding the PROGRAM.
2. Present the problem below for computer solution.

> Problem: Enter two numbers. Have the computer find the average (arithmetic mean) of the numbers.

3. Elicit the solution that follows.
    a) Analysis: In our first example today, we found the average as follows: $AV = \frac{70+80}{2} = \frac{150}{2} = 75$ Ask, "Can you describe this process in words?" Elicit that the average is the sum of the two numbers divided by two. Ask the students to use the legend that follows. Let A = 1st number, B = 2nd number, AV = average. (Ask, "Why not A?") Remind the students to ask themselves, "How do we find the average of the two numbers?" Elicit: Given two numbers A and B, AV = (A+B)/2 (Discuss A + B/2 vs. (A+B)/2)

b) Ask, "How do we *use* the algorithm to solve the problem?" Elicit the plan that follows.

Plan:

1) Put in two numbers: A, B.
2) Calculate the average: AV = (A+B)/2.
3) Print the average, AV.
4) End.

Ask the students to note that the computer uses the algorithm only *AFTER* the numbers are entered into memory cells A and B.

c) *Planned Output*: This should contain enough information so that whoever is using the program will understand what is going on.

```
RUN
THE AVERAGE OF TWO NUMBERS
     - BY JOE COOL
TELL ME YOUR TWO MARKS.
?90,70
YOUR AVERAGE IS 80
```

d) *The Program*:

```
10 PRINT "THE AVERAGE OF TWO NUMBERS"
20 PRINT "     - BY JOE COOL"
30 PRINT "TELL ME YOUR TWO MARKS."
40 INPUT A,B
50 AV = (A+B)/2
60 PRINT "YOUR AVERAGE IS";AV
70 END
```

APPLICATIONS:

Ask the students to make the necessary changes in the four stages above so that the computer will find the average of three numbers.

SUMMARY:

What is included in a complete computer solution to a problem?

HOMEWORK:

1. a) Show the output for the old program.
   b) Edit the old program so that it will produce the new output.

Old Program

```
10 INPUT X,Y,Z
20 A = X*Y*Z
30 PRINT A
40 END
```

New Output

```
RUN
FINDING THE PRODUCT OF THREE NUMBERS
WHAT ARE YOUR NUMBERS?
?10,2,5
YOUR PRODUCT IS 100
HAVE A GOOD DAY!
```

2. Complete the instructions for the problem which follows.
   - a) Develop an algorithm.
   - b) Write a plan.
   - c) Design an output.
   - d) Code a program.

Problem: Enter a number. Have the computer print 100 more than the number.

HOMEWORK SOLUTIONS:

1. a)

```
RUN
?10,2,5
100
```

b)

```
5  PRINT "FINDING THE PRODUCT OF THREE NUMBERS"
7  PRINT "WHAT ARE YOUR NUMBERS?"
10 INPUT X,Y,Z
20 A = X*Y*Z
30 PRINT "YOUR PRODUCT IS";A
35 PRINT "HAVE A GOOD DAY!"
40 END
```

2. a) Algorithm: X=N+100

b) Plan:
   1) Put in a number, N.
   2) Calculate: X=N+100.
   3) Print X.
   4) End.

c) Planned Output

```
RUN
100 MORE THAN YOUR NUMBER!
WHAT IS YOUR NUMBER?
?5
100 MORE IS 105
```

d) Program

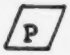
```
10 PRINT "100 MORE THAN YOUR NUMBER!"
20 PRINT "WHAT IS YOUR NUMBER?"
30 INPUT N
40 X = N+100
50 PRINT "100 MORE IS";X
60 END
```

Lesson: 14

AIM: How do we find the area of a square, a rectangle and a parallelogram?

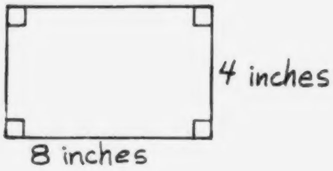
PERFORMANCE OBJECTIVES: The student will be able to...

1. find the area of a square and a rectangle;
2. define parallelogram;
3. find the area of a parallelogram, given the measure of its base and height.

VOCABULARY: parallelogram, base, height, area
SYMBOL: 

MOTIVATION:

1. Find the perimeter of this rectangle.
2. Find the area of this rectangle.
3. What do we mean by "area"? ...by "perimeter"?

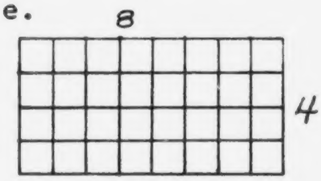


DEVELOPMENT:

1. Review the definition of area with the students.

   Definition: The area of a figure is the number of "unit squares" that can fit into the figure.

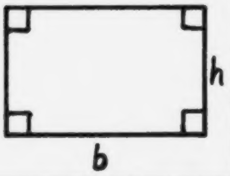
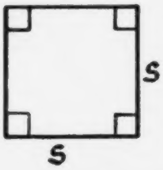
2. Hand out graph paper. Ask the students to draw a rectangle that is 8 units by 4 units.

   Elicit: To find the area of a rectangle, we multiply length and width or A = l·w.
   Point out that we also use the terms "base" and "height". Thus, the area of a rectangle is base·height or A = bh.
   Ask the students to note that the units are "square feet", "square inches", "square meters", ...

3. Ask the students to head a page and fill in the table that follows.

Perimeter and Area Formulas

Figure	Diagram	Perimeter	Area
1) Rectangle		$P = 2b + 2h$	$A = bh$
2) Square		$P = 4s$	$A = s^2$

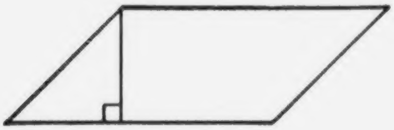
4. Using the graph paper, ask the students to draw each figure, calculate the perimeter and area and count the squares to verify the results.

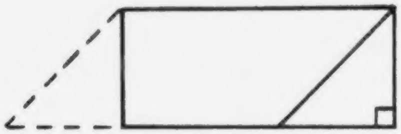
   a) square, $s = 3$ b) rectangle, $b = 7$, $h = 4$

5. Introduce a new figure... the parallelogram. Elicit the definition that follows.

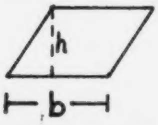
   Definition: A parallelogram (symbol ▱P) is a quadrilateral with two pairs of parallel sides.

   Ask, "How do we find the area of a parallelogram?" The teacher should display a physical model while the students cut out parallelograms from their graph paper. Ask the students to cut out a right triangle and insert it on the right to form a rectangle (as shown in the diagram below). Say to the students that the area of the rectangle can be found using the formula $A = bh$.





Since the area does not change when we move part of the figure to form a rectangle, the original parallelogram has the same area as the rectangle; e.g., $A = bh$. Ask the students to add this to their list of formulas.

3) Parallelogram		$P = 2b + 2h$	$A = bh$

6. a) Find the area of a /P/ with base 10 in. and height 4 in.
   b) What is the length of the base of a /P/ whose height measures 4 in. and whose area is 60 square in.?

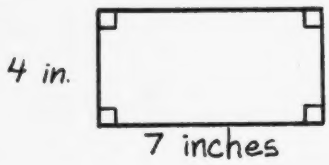
SUMMARY:

1. How do we find the area of a...
   a) square? b) rectangle? c) parallelogram?
2. What are the units for area?

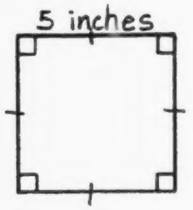
HOMEWORK:

In 1-6, find the area and perimeter.

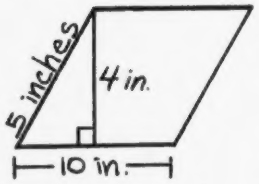
1.



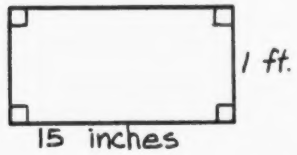
2.



3.



4.



5. A parallelogram with base 2 cm and height 3.5 cm.
6. A rectangle whose height is 15 units and whose base is twice its height.
7. Find the height of a rectangle whose area is 90 sq. in. and whose base is 18 in.
8. Find the base of a /P/ whose height is 3 in. and whose area is 12 sq. in.
9. For the problem, write
   a) an algorithm
   b) a plan
   c) a planned output and
   d) code the program.

Problem: Find the perimeter and area of a square whose length is given.

HOMEWORK SOLUTIONS:

1. A = 28 sq. in., P = 22 in.
2. A = 25 sq. in., P = 20 in.
3. A = 40 sq. in., P = 30 in.
4. A = 180 sq. in., P = 54 in.
5. A = 7 sq. cm, P = 11 cm
6. A = 450 sq. units, P = 90 units
7. 5 in.
8. 4 in.
9. a) Algorithm $P = 4S$, $A = S.S$ or $A = S^2$

   b) Plan: 1) Put in a value for the length of a side, S.
      2) Calculate the perimeter: $P = 4S$.
      3) Calculate the area: $A = S \times S$.
      4) Print A and P.
      5) End.

   c) Planned Output

```
PERIMETER AND AREA OF A SQUARE
TELL ME HOW LONG A SIDE IS.
?10
PERIMETER IS 40
AREA IS 100
```

   d) Program

```
10 PRINT "PERIMETER AND AREA OF A SQUARE"
20 PRINT "TELL ME HOW LONG A SIDE IS."
30 INPUT S
40 P = 4*S
50 A = S*S (or A = S↑2)
60 PRINT "PERIMETER IS";P
70 PRINT "AREA IS";A
80 END
```

Lesson: 15

AIM: How do we find the area of a triangle?

PERFORMANCE OBJECTIVES: The student will be able to...

find the area of a given triangle.

VOCABULARY: altitude, height, perpendicular

MOTIVATION:

Diagram 1

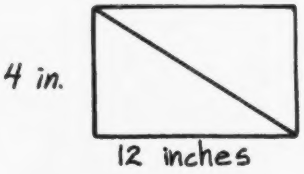
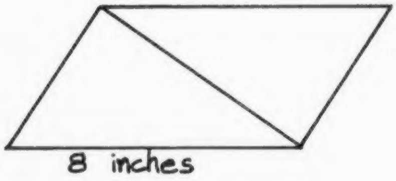


Diagram 2

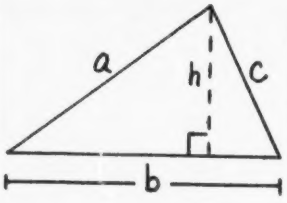


(height is 3 in.)

1. For diagram 1, find
   a) the perimeter,
   b) the area of the rectangle, and
   c) the area of each of the triangles formed.
2. For diagram 2, find
   a) the area of the parallelogram and
   b) the area of each of the triangles formed.

DEVELOPMENT:

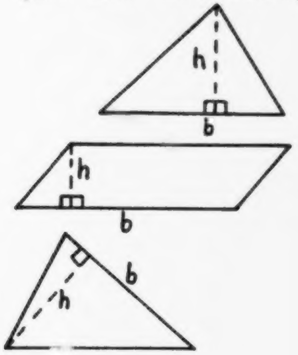
1. Ask, "What do these two examples illustrate?" Elicit that a triangle can be considered to be "half of a rectangle or parallelogram."
   Thus, the area of a triangle can be found by finding one-half the area of the rectangle or the ▱ that it "comes from." We therefore know that the area of a triangle = $\frac{1}{2}bh$. Add to yesterday's list of formulas.

4) Triangle		$P = a + b + c$	$A = \frac{1}{2}bh$

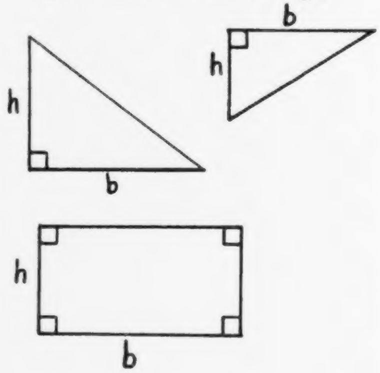
2. Ask, "What exactly is an altitude?" An altitude (or height) is a special line segment in a triangle that joins a vertex to a base, forming right angles with the base. We say, "An altitude is perpendicular to the base."

3. Ask the students to note that the altitudes may be located in one of three positions relative to the triangle.

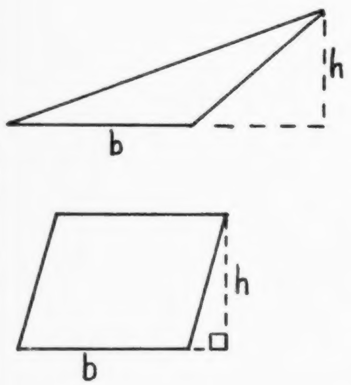
a) inside the figure (in the interior)



b) a side of the figure (on the boundary)



c) outside the figure (on the exterior)



APPLICATIONS:

1. Find the area of...
   a) a triangle with base 5 in. and altitude 4 in.
   b) a triangle with base 13 in. and altitude 5 in.
   c) a ▱P with base 4 in. and height 18 in.
   d) a rectangle with base 1 foot and height 8 in.
2. Find the base of a triangle whose area is 20 sq. in. and whose altitude is 10 in.
3. Find the area of a triangle with base 1 ft. and height 6 in.

SUMMARY:

How do we find the area of a triangle?

HOMEWORK:

1. Find the area of...
   a) a rectangle with base 5 ft. and height 3 ft.
   b) a P with base 3.5 in. and height 4.2 in.

c) a ▱P with base 2 ft. and height 4 in.
d) a triangle with base 6 in. and height 4 in.

2. Find the perimeter of the rectangle in 1a.

3. Find the area of a triangle with base 1.5 in. and height 6 in.

4. How long is the altitude if the base of a triangle is 6 in. and the area is 18 sq. in.?

5. For the problem, write:
   a) the algorithm
   b) the plan
   c) a planned output
   d) and a program.

   <u>Problem</u>: Find the area of a triangle given the base and height.

HOMEWORK SOLUTIONS:

1. a) 15 sq. ft.
   b) 14.7 sq. in.
   c) 96 sq. in.
   d) 12 sq. in.
2. 16 ft.
3. 4.5 sq. in.
4. 6 in.
5. a) <u>Algorithm</u> $A = \frac{1}{2}bh$
   b) <u>Plan</u>
      1) Put in values for the measures of the base and height: B,H.
      2) Calculate the area: A = .5*B*H or B*H/2.
      3) Print A.
      4) End.
   c) <u>Planned Output</u>

```
RUN
WHAT ARE THE BASE AND HEIGHT?
?10,4
THE TRIANGLE HAS AREA 20 SQUARE UNITS.
```

   d) <u>Program</u>

```
10 PRINT "WHAT ARE THE BASE AND HEIGHT?"
20 INPUT B,H
30 A = B*H/2
40 PRINT "THE TRIANGLE HAS AREA";A;"SQUARE UNITS."
50 END
```

Lesson: 16L

AIM: How do we solve computer problems involving areas of rectangles, parallelograms and triangles?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find a base (or a height) given the area and the height (or base) of a /P/, rectangle, or triangle;
2. write a complete computer solution to find the base or height of any geometric figure listed in 1.

VOCABULARY: base, altitude,height

MOTIVATION:

1. Find the length of the base of a rectangle with area 30 square units and whose height is 5 units.
2. Find the length of the base of a parallelogram with area 50 square units and whose height is 5 units.

DEVELOPMENT:

1. Elicit responses to the motivation examples as follows.
   1. $b = \frac{30}{5} = 6$ units   2. $b = \frac{50}{5} = 10$ units

   Say to the students that in these two problems, we are doing the reverse of what we are used to doing. Previously, most of our problems involved finding the area, if we knew the base and height. Now we are finding the base, if we know the area and the height. Let us do a computer solution for this.
2. Present the problem that follows for a computer solution.

Problem: Find the length of the base of a parallelogram if given the area and the height.

3. Elicit the following solution from the students. Remind the students to ask, "How do we find the length of the base when given the area and the height?

a) Analysis: Ask the students to examine the previous problems and conclude that if $A = BH$, $B = \frac{A}{H}$.

b) Develop the plan below with the students. Ask, "How do we <u>use</u> the algorithm to solve the problem?"

i) <u>Plan</u>:

1) Put in values for the area and the height: A,H.
2) Calculate the measure of the base: $B = \frac{A}{H}$.
3) Print B.
4) End.

ii) As you develop the plan with the students, ask the following questions.

1) What information is given for us to enter into the computer?
2) What algorithm do we want the computer to use in order to determine the length of the base?
3) In what memory cell will the answer to our problem be stored?

c) <u>Planned Output</u>:

```
RUN
FINDING THE LENGTH OF THE BASE
    OF A PARALLELOGRAM
ENTER THE AREA AND THE HEIGHT.
?50,5
THE BASE HAS LENGTH 10
```

Remind the students that the planned output should contain enough information so that whoever is using the program will understand what is going on. This includes a title, instructions to the user and an explanation of the results.

d) Ask the students to write the program that follows, enter it and run it. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you.

<u>Program</u>:

```
10 PRINT "FINDING THE LENGTH OF THE BASE"
20 PRINT "    OF A PARALLELOGRAM"
30 PRINT "ENTER THE AREA AND THE HEIGHT."
40 INPUT A,H
50 B = A/H
60 PRINT "THE BASE HAS LENGTH";B
70 END
```

APPLICATIONS:

1. Ask, "Can we use the above program for a rectangle?" Ask the students to:
   - a) enter the program and
   - b) edit it to find the base, given the area and height of a rectangle.
2. Ask the students to consider one of the following problems at their seats.
   - a) Enter the area and width of a rectangle. Have the computer find the length and the perimeter of the rectangle.
   - b) Enter the area of a triangle, also the height. Find the base. $(B = \frac{2A}{H})$
3. Send the students to the computer only after they finish the four steps of a computer solution. Ask the students to put models on the chalkboard.

SUMMARY:

In order for the computer to do any of the programs discussed in today's lesson, what is the most important stage of computer programming? (the analysis)

HOMEWORK:

1. Find the height of a /P/ with area 75 square units and base 15 units.
2. Find the length of the base of a rectangle if the area is 60 square units and the height is 5 units.
3. What is the perimeter of a rectangle with area 100 square units and base 20 units?
4. Find the height of a triangle with base 4.5 units and area 9 square units.
5. For the problem, write...
   - a) the algorithm
   - b) the plan
   - c) a planned output
   - d) and a program.

__Problem__: Enter the length of the side of a square and the height of a rectangle with the same area as that of the square. Find the base of the rectangle.
(__Hint__: First try this with pencil and paper;.e.g. the square has length 10 units and the height of the rectangle is 25 units.)

HOMEWORK SOLUTIONS:

1. 5 units
2. 12 units
3. 50 units
4. 4 units
5. a) __Algorithm__

   A = SxS

   B = A/H

   b) __Plan__

   1) Enter values for the side of the square and the height of the rectangle: S,H.
   2) Calculate the area of the square: A = SxS.
   3) Calculate the length of the base: B = A/H.
   4) Print B.
   5) End.

   c) __Planned Output__:

```
RUN
EQUAL AREAS
HOW LONG IS THE SIDE OF THE SQUARE?
?10
HOW LONG IS THE HEIGHT OF THE RECTANGLE?
?25
THE BASE OF THE RECTANGLE HAS LENGTH 4
```

   d) __Program__

```
10 PRINT "EQUAL AREAS"
20 PRINT "HOW LONG IS THE SIDE OF THE SQUARE?"
30 INPUT S
40 PRINT "HOW LONG IS THE HEIGHT OF THE RECTANGLE?"
50 INPUT H
60 A = S*S
70 B = A/H
80 PRINT "THE BASE OF THE RECTANGLE HAS LENGTH";B
90 END
```

Lesson: 17L

AIM: How do we analyze and code a program to determine the cost of carpeting a rectangular room?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the area of a rectangular room;
2. determine the cost of carpeting such a room;
3. analyze the problem presented in 1 and 2 and code a program.

VOCABULARY: dimensions

MOTIVATION:

1. Carpeting is sold by the square yard. A rectangular room has dimensions of 15 feet by 9 feet. We want to have carpeting put down in this room. How many square yards of carpeting is needed?
2. If carpeting costs $20 a square yard, what will be the cost of carpeting the above room?

DEVELOPMENT:

1. Ask the students to solve the motivation problems as follows.
   a) Change the feet to yards. $\frac{15}{3}$ = 5 yd. and $\frac{9}{3}$ = 3 yd.
   b) Compute the area (expressed in square yards).
      $A = l.w$
      $A = 5 \cdot 3$
      $A = 15$ sq. yd.
   c) Compute the cost of the carpeting.
      $C = 15 \text{ sq. yd.} \times \frac{\$20}{\text{sq. yd.}} = \$300$
2. Present the problem below for a computer solution.

Problem: Given the length and width (in yards) and the cost of the carpet per square yard, calculate the cost of carpeting a rectangular room.

3a. Before the students begin to formulate a solution, ask, "How is the problem we will be solving on the computer different from the one that we just solved with pencil and paper?" Elicit that the dimensions here will be entered in yards.

3b. Analysis

i) Ask, "How do we find the area of a rectangular room if we know the length and width?" Elicit A = L.W where
memory cell A stores the area (expressed in sq. yd.),
memory cell L stores the length (expressed in yd.),
and memory cell W stores the width (expressed in yd.)

ii) Ask, "How do we find the cost of the carpeting?" Elicit C = A.P where
memory cell C stores the cost,
memory cell A stores the area (in sq. yd.), and
memory cell P stores the price (per sq. yd.).

3c. Ask, "How do we use the algorithm to solve the problem?"

Plan:

1) Enter values for the length, width and price: L,W,P.
2) Calculate the area: A = L.W.
3) Calculate the cost: C = A.P.
4) Print C.
5) End.

As the students develop the plan, ask the following questions.

i) What information must be entered into the computer?
ii) What algorithm will the computer use to determine the area? ...the cost?
iii) What value do we want displayed as the solution to our problem? In what memory cell is the value located?

c) Ask the students to design a planned output as follows.

Planned Output:

```
RUN
STEVE'S CARPETS
ENTER THE LENGTH AND WIDTH OF YOUR ROOM (IN YARDS).
?5,3
```

```
WHAT IS THE PRICE PER YARD OF YOUR CARPET?
?20
THIS CARPET WILL COST $1300
```

Remind the students of the importance of the title, the instructions to the user, and an explanation of the results.

d) Ask the students to code the program that follows. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you.

The Program:

```
10 PRINT "STEVE'S CARPETS"
20 PRINT "ENTER THE LENGTH AND WIDTH OF YOUR ROOM (IN YARDS)."
30 INPUT L,W
40 PRINT "WHAT IS THE PRICE PER YARD OF YOUR CARPET?"
50 INPUT P
60 A = L*W
70 C = A*P
80 PRINT "THIS CARPET WILL COST $";C
90 END
```

APPLICATIONS:

After running the above program, ask the students to modify the program so that the length and width can be entered in feet; e.g., have the computer divide the measure of each dimension by 3 to calculate the number of yards.

SUMMARY:

How is the cost of carpeting a rectangular room calculated?

HOMEWORK:

Find the cost of carpeting each rectangular room:

1. 8 yards by 4 yards at $15 per square yard
2. 10 yards by $2\frac{1}{2}$ yards at $18 per square yard
3. 24 feet by 9 feet at $14.99 per square yard
4. 15 feet by 10 feet at $12.00 per square yard
5. Find the base of a /P/ with area 55 square units and height 5 units.

6. Find the height of a triangle with area 12 square units and base 4 units.

7. Write...
   a) the algorithm
   b) the plan
   c) a planned output and
   d) the program.

<u>Problem</u>: After the cost of carpeting a rectangular room is calculated, a $150 installation fee is to be added to the cost. Calculate the entire cost of installing a carpet, given the dimensions of the room (in yards) and price of the carpet per square yard.

HOMEWORK SOLUTIONS:

1. $480
2. $450
3. $359.76
4. $200.00
5. 11 units
6. 6 units

7. a) <u>Analysis</u>:

   A = L·W
   C = A·P
   T = C + 150 (T = total cost)

   b) <u>Plan</u>:

   1) Put in the length, width and price per square yard: L,W,P.
   2) Calculate the area (in sq. yds.): A = L.W.
   3) Calculate the cost: C = A.P.
   4) Calculate the total cost: T = C+150.
   5) Print T.
   6) End.

   c) <u>Planned Output</u>:

```
TELL ME THE LENGTH (IN YD.) AND WIDTH (IN YD.) AND
PRICE (PER SQ. YD.).
?8,4,15
YOUR COST IS $630
```

   d) <u>Program</u>:

```
10 PRINT "TELL ME THE LENGTH (IN YD.), WIDTH (IN YD.)"
20 PRINT "PRICE (PER SQ. YD.)."
30 INPUT L,W,P
40 A = L*W
50 C = A*P
60 T = C + 150
70 PRINT "YOUR COST IS $";T
80 END
```

Lesson: 18L

AIM: How do we code a program to examine the effects of changing A, b or h in the formula A = BH?

PERFORMANCE OBJECTIVES: The student will be able to...

1. state the effect of changing A, B or H in the formula A =BH;
2. code a program to study the effects of changing one variable.

VOCABULARY: area, base, height, doubled, halved, tripling, constant, squaring, variable

MOTIVATION:

1. How does the area of a rectangle change if the base is doubled and the height remains the same?
2. How does the base of a rectangle change if the height is doubled and the area remains the same?

DEVELOPMENT:

1. Ask the students to build a chart for the first motivation question as follows.

B	H	A
4	3	12
8	3	24
16	3	48
32	3	96

Ask the students to state that the area also doubles.

2. Similarly, elicit a chart for the second motivation question; e.g.,

B	H	A
24	2	48
12	4	48
6	8	48
3	16	48

To construct the chart, ask the students to begin with B=24 and A=48. Fill in 48 for the area in all four rows and then ask the students to determine the height. Ask them to conclude that the length of the base is halved.

3. Say to the students that although the computer can help us build charts as we just did, we need the human brain to make judgements and recognize patterns. Say that we will use the computer this period to assist us in our study of the formula A = BH where A represents the area of the rectangle, B represents the measure of the base of the rectangle, and H represents the measure of the height of the rectangle.

4. Divide the students into three groups. Assign the following tasks to the groups.

   For the problem you are given, complete the instructions.

   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan showing how to use the algorithm to solve the problem.
   c) Design a planned output.
   d) Code a program
   e) RUN the program and try...
      i) doubling the changing variable;
      ii) tripling the changing variable;
      iii) halving the changing variables.

   Develop charts for i, ii and iii as we did in the earlier examples we solved today.

   Group 1

   > Problem: Write a program to keep the measure of the base of a rectangle constant (for example, always 4 or always 5), but allow the height to change. Have the computer calculate the measure of the area and then display values for the base, height and area (all three variables, B, H and A).

   Group 2

   > Problem: Write a program to keep the measure of the height of a rectangle constant (for example, always 4 or always 5), but allow the measure of the base to

change. Have the computer calculate the area and then display values for the base, height and area (all three variables, B, H and A).

Group 3

Problem: Write a program to keep the area of a rectangle constant (for example, always 48 or always 100), but allow the measure of the base to change. Have the computer calculate the area and print all three variables and then display values for the base, height and area (all three variables, B, H and A).

5. Ask each group to record their results (charts) on the chalkboard. Ask the students, "What conclusions can we make for each chart?"

Applications: Ask the students to use their programs to compile data to show that there is no discernable pattern for squaring the changing variable or for adding seven to the changing variable.

SUMMARY:

1. How can the computer be used in making conclusions about the effects of changing one variable in a formula?
2. Can we always predict the change that will occur in one variable as a result of changing another variable in a formula?
3. Do the conclusions drawn in today's lesson apply to any other geometric figure that we have studied? Why?

HOMEWORK:

1. Find the area and perimeter of a rectangle with base 7.2 units and height 3.4 units.
2. Find the area, expressed in square inches, of a parallelogram with base 12 feet and height 13 inches.
3. Find the base of a parallelogram with area 72 square units and height 12 units.

4. A rectangular living room, 18 feet by 12 feet, is to be carpeted with a rug that costs \$37.92 per square yard. How much will it cost to buy this rug?
5. The area of a triangle is 24 square units and its base measures 12 units.
   a) Find the height of the triangle.
   b) Create a chart to show what happens to the measure of the height as the measure of the base is halved and the area remains 24. Write your conclusion in words.

HOMEWORK SOLUTIONS:

1. A = 24.48 square units, P = 21.2 units
2. 1872 sq. in.
3. 6 units
4. \$910.08
5. a) h = 4 units

   b)

B	H	A
12	4	24
6	8	24
3	16	24
$1\frac{1}{2}$	32	24

As the measure of the base is halved, the measure of the height doubles.

Lesson: 19

AIM: What is the binary (base two) numeration system and how does it compare with the decimal (base ten) numeration system?

PERFORMANCE OBJECTIVES: The student will be able to...

1. change numbers written as numerals in other bases to decimal numerals;
2. state the advantages and disadvantages of using the binary vs. the decimal system.

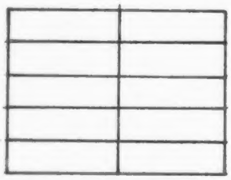
VOCABULARY: binary, base number, numeration system, decimal system, base two, base ten, place value

MOTIVATION:

How can someone claim he/she has a ten dollar bill on him/her when all he/she has is a five dollar bill and three singles?

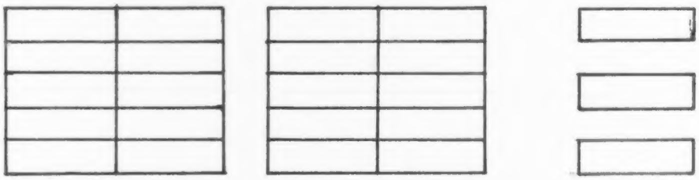
DEVELOPMENT:

1. In our usual meaning for "10", we are grouping by 10's. Thus, 10 means one group of ten and no ones and



One group of ten and no ones

23 means two groups of ten and three ones.

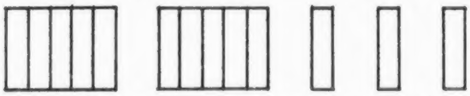


Two groups of ten and three ones

Definition: A system that groups by tens is called a decimal or base ten numeration system.

2. In base five, however, we would group by fives.

   Thus,



   two groups of fives and three ones would be written, 23_5.

   Ask the students to note that in base ten, besides grouping by tens, there are also exactly ten symbols: 0,1,2,3...9, while in base five there are five symbols: 0,1,2,3,4.

   Therefore, when our friend said he (or she) had a ten dollar bill, he could have been referring to his 10_5 bill or $5 bill.

3a. Say to the students that a very convenient and useful base in computer work is the base two or binary system. Ask, "How do we group in base two? How many symbols would you expect to find in base two?" Elicit that here the only two symbols are 0,1.

3b. Build up the chart with the students:

(Base ten)	(Base two)
0	0
1	1
2	10
3	11
4	100
5	101
6	110

4. Ask the following questions as you develop the chart.
   - i) What symbol is used for zero in base ten? ...in base two?
   - ii) What symbol is used for one in base ten? ...in base two?
   - iii) What symbol(s) is (are) used for two in base ten? ...in base two? Say that since there are no more symbols left, we form a new group of two. (You might want to illustrate this with rectangle models.)

iv) What symbol(s) is (are) used for three in base ten? ...in base two?

v) What symbol(s) is (are) used for four in base ten? ...in base two? Say that since there are no more symbols to place in the twos group or place, we form a new group containing two groups of two or four.

vi) What symbol(s) is (are) used for five in base ten? ...in base two?

vii) What symbol(s) is (are) used for six in base ten? ...in base two?

5. Ask students to compare the place values in base ten to those in base two; e.g.,

Base ten						Base two					
10000's	1000's	100's	10's	1's		32's	16's	8's	4's	2's	1's
4	2	5	3	1	vs.	1	1	0	0	1	0

Thus, 110010_2 represents the number

1x32 = 32
1x16 = 16
0x8 = 0
0x4 = 0
1x2 = 2
1x0 = $\underline{0}$
50_{10} or 50.

Remind the students that when writing base ten numerals, we usually do not indicate the base.

6. Ask the students to generalize that to change a number expressed as a base two (binary) numeral into a base ten (decimal) numeral, they will add the numbers naming the place values above the ones in the numeral; e.g.,

	8's	4's	2's	1's	
1101_2	1	1	0	1	1
					4
					$\underline{8}$
					13

1101_2 represents the number thirteen, written 13 in base ten.

Applications: Change each number, which is written in base two to base ten.

a) 110_2
b) 11011_2
c) 1111_2
d) 11111_2
e) 1000_2
f) 1001_2

SUMMARY:

1. How do you change a base two numeral into a base ten numeral?
2. Which system, base two or base ten, uses fewer symbols to express each number?
3. What are the disadvantages of the binary system?
4. In terms of computers, why is the use of only two symbols an advantage?

HOMEWORK:

Change each of the following to a base 10 numeral.

1. 1010_2
2. 11111_2
3. 110_2
4. 1001_2
5. 10000_2
6. 111000_2
7. Find the cost of carpeting a 12' by 9' den using a carpet that costs \$15 per square yard.
8. Find the area of a rectangle with base 12 units and height 10 units.
9. Find the area of a triangle with base 5 units and height 4 units.
10. Build up a chart to show what happens to the area of a parallelogram as the measure of the base is doubled but the measure of the height remains the same. State your conclusions in words.
11. Evaluate 3 + 6 x 2 + 7.

12. If A = 6 and B = 3, what is the result of the computer executing each statement?

a) `10 PRINT "A=6"`
b) `30 PRINT "A=",A`
c) `50 PRINT "B=3"`
d) `40 PRINT "B=";3`
e) `90 PRINT "A=";A;"B=";B`

HOMEWORK SOLUTIONS:

1. 10
2. 31
3. 6
4. 9
5. 16
6. 56
7. $180
8. 120 sq. units
9. 10 sq. units

10.

B	H	A
3	5	15
6	5	30
12	5	60
24	5	120

As the measure of the base doubles, the area also doubles.

11. 22

12.
```
a) A = 6
b) A =       6
c) B = 3
d) B = 3
e) A = 3 B = 6
```

Lesson: 20

AIM: How do we code a program to convert binary numerals into decimal numerals?

PERFORMANCE OBJECTIVES: The student will be able to...

1. convert from base two to base ten;
2. write a program to perform the conversion.

VOCABULARY: conversion, binary, decimal, place value, value

MOTIVATION:

Convert each to a base ten numeral.

1. 1100_2
2. 1111_2
3. 1001_2

DEVELOPMENT:

1a. Write the responses to the motivation examples as follows.

8's	4's	2's	1's
1	1	0	0_2

8x1 = 8
4x1 = 4
2x0 = 0
1x0 = 0
12

8's	4's	2's	1's
1	1	1	1_2

8x1 = 8
4x1 = 4
2x1 = 2
1x1 = 1
15

8's	4's	2's	1's
1	0	0	1_2

8x1 = 8
4x0 = 0
2x0 = 0
1x1 = 1
9

1b. Ask the following questions, as you write the responses.

- i) What is the place value of the left most digit?
- ii) How do we find the value of that digit?
- iii) What is the place value of the second digit from the left?

iv) How do we find the value of that digit?

v) What is the place value of the third digit from the left?

vi) How do you find the value of that digit?

vii) What is the place value of the digit on the right?

viii) How do we find the value of that digit?

ix) What do we have to do with our products to complete the conversion?

2. Present the following problem for a computer solution.

> Problem: Enter the digits of a number written as a four-digit base two numeral. Enter one digit at a time, starting with the left most digit. Have the computer convert to base ten. (Note: 10_2 will be entered as 0,0,1,0.)

3. Elicit the solution that follows from the students.

a) Analysis Ask, "If $ABCD_2$ is our base two numeral, how do we make the conversion?" Elicit what follows.

8's	4's	2's	1's
A	B	C	D_2

8xA
4xB
2xC
1xD

N = 8xA + 4xB + 2xC + D

where memory cell N stores the number written as a base ten numeral. Ask, "Why is the multiplication of D by 1 unnecessary?"

b) Ask, "How do we use the algorithm to solve the problem?" Elicit the plan that follows.

Plan

1) Put in values for each digit from left to right: A,B,C,D.

2) Calculate the base ten conversion: N=8xA+4xB+2xC+D.

3) Print N.

4) End.

Ask the following questions as you develop the plan with the students.

i) What information do we need to enter into the computer?

ii) What algorithm do we want the computer to use in order to calculate the base ten conversion.

iii) What do we want the computer to display? In what memory cell is it stored?

c) Ask the students to design a planned output as follows.

Planned Output:

```
CONVERTING BINARY TO DECIMAL
ENTER A 4-DIGIT NUMERAL,
ONE DIGIT AT A TIME BEGINNING ON THE LEFT.
?1,1,0,1
IN BASE TEN, THE NUMBER IS WRITTEN AS 13
```

Point out to the students the importance of the title, the instructions to the user and an explanation of the results.

d) Ask the students to code the program that follows.

The Program:

```
10 PRINT "CONVERTING BINARY TO DECIMAL"
20 PRINT "ENTER A 4-DIGIT BINARY NUMERAL,"
30 PRINT "ONE DIGIT AT A TIME BEGINNING ON THE LEFT."
40 INPUT A,B,C,D
50 N = 8*A + 4*B + 2*C + D
60 PRINT "IN BASE TEN, THE NUMBER IS WRITTEN AS";N
70 END
```

SUMMARY:

How do we convert numbers written in base two to base ten?

HOMEWORK:

Change each of the following to a base ten numeral.

1. 1100_2
3. 101010_2
2. 11100_2
4. Find the cost of carpeting a 10 ft. x 9 ft. den with carpet that costs $41.23 per square yard.
5. Revise today's program to change a number expressed as a 6-digit binary numeral into a decimal numeral.

HOMEWORK SOLUTIONS:

1. 12
2. 28
3. 42
4. $412.30
5.

```
10 PRINT "CONVERTING BINARY TO DECIMAL"
20 PRINT "ENTER A 6-DIGIT BINARY NUMERAL,"
30 PRINT "ONE DIGIT AT A TIME BEGINNING ON THE LEFT."
40 INPUT A,B,C,D,E,F
50 N = 32*A + 16*B + 8*C + 4*D + 2*E + F
60 PRINT "IN BASE TEN, THE NUMBER IS WRITTEN AS";N
70 END
```

Lesson: 21

AIM: How do we code a program to display and add all possible numbers that can be formed using three different nonzero digits?

PERFORMANCE OBJECTIVES: The student will be able to...

1. list all six permutations of a number written as a three-digit numeral (with different digits);
2. code a program using the above idea.

VOCABULARY: permutation, sum, palindrome

MOTIVATION:

Using the digits 6, 5 and 9 only once, write all possible 3-digit numbers.

DEVELOPMENT:

1. Ask the students to list their responses to the motivation problem as follows:
   659, 695, 956, 569, 965, 596
2. Say to the students that the idea of rearranging objects has always intrigued people. Often they "wheel" their bets at the race track or in the lottery by just rearranging a few digits.
3. State the definition of permutation for the students.
   Definition: A permutation of objects is a (re)arrangement of the objects in a definite order.
   Example 1: Find all permutations of the digits 2,8,1.
   Answer: 281, 218, 182, 821, 128, 812
   Ask the students to notice that with 3 objects, we have 6 permutations.
   Example 2: With 3 spaces on a shelf and 3 books (Algebra, Biology, Computer), there are 6 permutations or arrangements possible. ABC, ACB, BAC, BCA, CAB, CBA
4. Present the problem that follows for a computer solution.

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Problem: Enter three nonzero digits. Have the computer display all possible 3-digit numbers that can be written using each of the three digits only once. Also, have the computer display the sum of possible numbers.

5. Ask the students to solve the problem as follows.

   a) Analysis:

      i) Ask, "If the three digits are A, B and C, how do we get the computer to determine the number that will be written as ABC?" Ask the students to recall the work they did yesterday and use it as a model; e.g.,

100's	10's	1's
A	B	C

100xA
10xB
1xC

N1 = 100xA + 10xB + C

where memory cell N1 stores the first possible permutation.

      ii) Ask, "What algorithms can be used to have the computer obtain the other five permutations?"

Elicit N2 = 100xB + 10xA + C
N3 = 100xA + 10xC + B
N4 = 100xC + 10xA + B
N5 = 100xC + 10xB + A
N6 = 100xB + 10xC + A

      iii) Ask, "How do we get the sum?" Elicit S = N1 + N2 + N3 + N4 + N5 + N6.

      iv) Ask the students to use their algorithms with A=5, B=2 and C=1 to verify that they are getting all possible permutations.

   b) Ask the students to develop the plan that follows.

Plan:

1) Put in values for three nonzero numbers: A,B,C.
2) Calculate the six permutations:

N1 = 100xA + 10xB + C
N2 = 100xB + 10xA + C
N3 = 100xA + 10xC + B

N4 = 100xC + 10xA + B

N5 = 100xC + 10xB + A

N6 = 100xB + 10xC + A

3) Calculate the sum: S = N1+N2+N3+N4+N5+N6.
4) Print N1, N2, N3, N4, N5, N6, S.
5) End.

Ask the following questions as the students write the plan.

i) What information must be entered into the computer?
ii) What algorithm do we want the computer to use in order to determine each permutation?
iii) What algorithm do we want the computer to use in order to determine the sum?
iv) What do we want the computer to display? In what memory cells are these values stored?

c) Ask the students to design a planned output as follows.

Planned Output:

```
FORMING ALL POSSIBLE 3-DIGIT NUMBERS
USING 3 DIGITS
WHAT DIGITS DO YOU WANT TO USE?
?5,2,1
THE NUMBERS ARE:
521   251   512   152   125   215
THEIR SUM IS 1776
```

d) Ask the students to code the program that follows.

The Program:

```
10  PRINT "FORMING ALL POSSIBLE 3-DIGIT NUMBERS"
20  PRINT "USING 3 DIGITS"
30  PRINT "WHAT DIGITS DO YOU WANT TO USE?"
40  INPUT A,B,C
50  N1 = 100*A + 10*B + C
60  N2 = 100*B + 10*A + C
70  N3 = 100*A + 10*C + B
80  N4 = 100*C + 10*A + B
90  N5 = 100*C + 10*B + A
100 N6 = 100*B + 10*C + A
110 S = N1 + N2 + N3 + N4 + N5 + N6
120 PRINT "THE NUMBERS ARE:"
130 PRINT N1; N2; N3; N4; N5; N6
140 PRINT "THEIR SUM IS";S
150 END
```

On some computers, no spaces will be left between displayed numbers. Adjust the planned output and the program accordingly.

APPLICATION:

1. For the problem below, execute the following instructions.
   a) Write an analysis of the problem that leads to an algorithm.
   b) Write a plan that outlines how you use the algorithm to solve the problem.
   c) Design a planned output.
   d) Code a program.

   Problem: Enter three nonzero digits. Have the computer display the...
   a) number formed by the digits;
   b) number formed when the digits are reversed;
   c) sum of the two numbers.
2. A palindrome is a number whose value is the same when read left to right as it is when read right to left; e.g., 121 or 333. A palindrome is sometimes produced by adding a number to its "reverse." Trace the program written in 1d using each triple of digits that follow. Which triple (s) produce a palindrone when the number formed by it is added to its "reverse"?
   a) 1,5,6
   b) 3,2,6
   c) 3,4,5

SUMMARY:

What do we mean by a permutation of objects?

HOMEWORK:

Change each to a base ten numeral.

1. 110101_2
2. 1111111_2
3. 10000000_2
4. 24_5
5. 100_5

6. Which is the more expensive room to carpet,
   a room 4 yds. by 3 yds. at $15.25 per sq. yd. or
   a room 5 yds. by 4 yds. at $13.75 per sq. yd.?

7. List all permutations of the letters N,O (using 2 at a time).

8. List all permutations of the digits 5,2,7,1 using all four at a time.

9. Rewrite today's program to produce this planned output.

Planned Output

```
TELL ME THREE DIFFERENT DIGITS.
?5,2,1
THESE ARE THE POSSIBLE 3-DIGIT NUMBERS I CAN MAKE:
521     251     512     152     125     215
THEY ADD UP TO 1776
THANK YOU!
```

HOMEWORK SOLUTIONS:

1. 53   2. 127   3. 128   4. 14   5. 25

6. The first room will cost $183; the second room will cost $275. Even though a more expensive type of carpet is used for the first room, the second room is a more expensive room to carpet.

7. NO, ON

8.
```
5271    2571    7251    1572
5721    2751    7521    1752
5712    2715    7512    1275
5172    2175    7152    1725
5217    2517    7215    1527
5127    2157    7125    1257
```

9. Program

```
10 PRINT "TELL ME THREE DIFFERENT DIGITS"
20 INPUT A,B,C
30 N1 = 100*A + 10*B + C
40 N2 = 100*B + 10*A + C
50 N3 = 100*A + 10*C + B
70 N5 = 100*C + 10*B + A
80 N6 = 100*B + 10*C + A
90 S = N1 + N2 + N3 + N4 + N5 + N6
100 PRINT "THESE ARE THE POSSIBLE 3-DIGIT NUMBERS I CAN MAKE:"
110 PRINT N1; N2; N3; N4; N5; N6
120 PRINT "THEY ADD UP TO";S
130 PRINT "THANK YOU!"
140 END
```

On some computers, no spaces will be left between displayed numbers. Adjust the planned ouput and the program accordingly.

Lesson: 22

AIM: What are the different ways people get paid?

PERFORMANCE OBJECTIVES: The student will be able to...

1. state the various methods by which people get paid;
2. compute the amount of pay based on hourly, weekly, or annual salaries;
3. compute the amount of pay based on given rates of commission.

VOCUABULARY: rate per hour, commission, wages, annual

MOTIVATION:

Matt washes cars in his neighborhood to earn pocket money. If he charges $3.50 per car, how much money will he earn in a month in which he washes 16 cars?

DEVELOPMENT:

1. Elicit 16 x $3.50 = $56.00 as a response for the motivation problem.
   Ask, "If Tony cleans tables in a fast-food restaurant, do you think he gets paid an amount of money for each table?" Elicit that Tony probably is paid based on how many hours he works each week.
2. Say that some workers are paid based on how many times they perform a task; for example, $3.50 per car washed, $10.00 per lawn mowed, $10,000 per concert.
   Ask, "How can we determine the pay for 8 cars washed at $3.50 per car? ...4 lawns mowed at $10 per lawn? ...24 concerts at $10,000 per concert?"
   (8 x 3.50 = $28.00, 4 x 10 = $40, 24 x 10,000 = $240,000)
   Say that in general, Pay = (Number of times) x (Rate per time).
3. Discuss with the students that many workers are paid based on how much they work; some per hour, some per week and some per year

Ask, "If Tony is paid $4.00 hourly, how can we find his pay for the week?"

Elicit that Pay = (Number of hours) x (Rate per hour)

Say that if Tony works $16\frac{1}{2}$ hours during the week at $4.00 per hour, he earns $(16\frac{1}{2}) \times (4.00) =$ $66.00.

Ask the students to complete the exercises that follow.

Exercises: Find the wages earned at...

a) $3.00 per hour for 6 hours.
b) $4.25/hr for 5 hours.
c) $5.24 per hour for 8 hours.

4. Say that salespeople are often paid a percentage of the amount they sell. This is called a commission. For example, Jeremy receives an 8% rate of commission for selling automobiles. How much will he receive if his sales are $16,000?

   Elicit that Commission = (Rate)x(Amount of Sales).

   Ask the students to write 8% as a decimal numeral,

   Commission = .08 x $16,000 = $1,280.00

APPLICATIONS:

1. Compute the amount of pay for...
   a) 6 rooms painted at $25.00 per room.
   b) 14 tables assembled at $3.75 per table.
   c) 17 hours at $4.83 per hour.
   d) 15 hours at $5.08 per hour.
   e) $12\frac{1}{2}$ hours at $4.80 per hour.
   f) 52 weeks at $325 per week.

2. Find the amount of commission for...
   a) rate = 12%, sales = $3,600.
   b) rate = 15%, sales = $4,620.
   c) rate = 9%, sales = $1,624.
   d) rate = $37\frac{1}{2}$%, sales = $24,000.

3. Who earns more and by how much...
   a) Jane, at $4.50 per hour for 40 hours, or Joan at $195 per week?
   b) Mark, at $350 per week, or Bryan at $1,400 per month, if we compute the yearly pay for each?

c) a salesman earning $28,000 per year, or a saleswoman earning 11% on sales of $270,000?

SUMMARY:

1. What are the ways that people get paid?
2. How can we compute the pay for a worker paid hourly? ...for a salesperson on commission?

HOMEWORK:

1. Find the area of a rectangle with a base of 12" and a height of 9".
2. Find the area of a triangle with a base of 4" and a height of 8".
3. Express 1011_2 as a decimal numeral.
4. Compute the pay for each of the following.
   - a) 6 lawns at $11.50 per lawn.
   - b) 18 hours at $5.24 per hour.
   - c) 1 year at $425 per week.
5. Compute the commission for each of the following.
   - a) rate = 3%, sales = $4,375.
   - b) rate = 18%, sales = $2,058.
   - c) rate = $7\frac{1}{2}$%, sales = $14,072.
6. Find the weekly pay if the annual salary is...
   - a) $10,400.
   - b) $9,802.
7. Rachel works 32 hours at $5.25 per hour. How many hours must Randy work at $4.20 per hour to earn the same pay?

HOMEWORK SOLUTIONS:

1. 108 sq. in.

2. 16 sq. in.

3. 11

4. a) $69
   b) $94.32
   c) $22,100

5. a) $131.25
   b) $370.44
   c) $1055.40

6. a) $200
   b) $188.50

7. 40 hours

Lesson: 23L

AIM: How do we code a program to determine salary?

PERFORMANCE OBJECTIVES: The student will be able to:

1. develop an algorithm to determine a salary;
2. formulate a plan for a computer solution of the problem of determining a salary;
3. design an output for a program to determine a salary;
4. develop a program to determine a salary.

VOCABULARY: rate per hour, salary, annual, wages

MOTIVATION:

Jeremy earns $2.75 per hour babysitting for his next door neighbors. He is paid every four weeks. How much does he earn in a four- week period where he works five hours the first week, seven hours the second week, three hours the third week, and nine hours the fourth week?

DEVELOPMENT:

1. Ask the students to suggest solutions for the motivation problem as follows.

Solution 1:

First week	$2.75 x 5 =	$13.75
Second week	$2.75 x 7 =	19.25
Third week	$2.75 x 3 =	8.25
Fourth week	$2.75 x 9 =	+24.75
		$66.00

Solution 2:

First week	5	$2.75 x 24 = $66.00
Second week	7	
Third week	3	
Fourth week	+9	
	24 hours total	

2. Ask, "If we want the calculations to be performed by the computer, what tasks must we do in preparing a computer solution?"

Elicit:

- a) Develop an algorithm for finding the salary.
- b) Formulate a plan showing how to use the algorithm to solve the problem.
- c) Design an output.
- d) Write the program.

3. Present the following problem for a computer solution.

Problem: Enter the number of hours worked and the hourly rate of pay. Have the computer determine the weekly salary.

4. Ask, "What calculations must be done to compute weekly salary?" Elicit the algorithm:

Weekly salary = Hours worked x Rate per hour

Ask the students to set up a legend identifying what each memory cell stores as follows.

WS = weekly salary
H = hours worked
R = rate per hour

5a. Ask the students to formulate the plan that follows.

Plan:

1) Enter values for the number of hours and the hourly rate of pay: H,R.
2) Calculate the weekly salary: $WS = H \cdot R$.
3) Print WS.
4) End.

5b. Ask the following questions as you develop the plan with the students.

- i) What information do we need to enter into the computer?
- ii) What algorithm do we want the computer to use in order to calculate the weekly salary?
- iii) What value do we want the computer to display? In which memory cell is this stored?

6. Ask the students to make suggestions on a design for a planned output.

Planned Output

```
FINDING WEEKLY WAGES
ENTER HOURS WORKED AND RATE PER HOUR.
?40,6.25
THE SALARY IS $250
```

7a. Stress the importance of the title, the instructions to the user and an explanation of the results. Ask, "Which statements in BASIC are needed for our program?"
Elicit Program I:

```
10 PRINT "FINDING WEEKLY WAGES"
20 PRINT "ENTER HOURS WORKED AND RATE PER HOUR."
30 INPUT H,R
40 WS = H*R
50 PRINT "THE SALARY IS $"; WS
60 END
```

(Note: Amounts of money must be entered without "$" signs in the input mode.)

7b. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

7c. Circulate around the laboratory assisting students as needed.

7d. Group the students who are unable to write the program. Work through some of the coding with the students.

7e. Ask the students who complete the problem presented to solve the problems presented in the Applications.

APPLICATIONS:

1. Revise Program I so that it will produce the following output.

   Planned Output

```
COMPUTING WAGES
WHAT IS THE NUMBER OF HOURS
?37.5
WHAT IS THE HOURLY RATE
?6
THE WAGES ARE $225
```

2. Modify Program I by creating your own planned output.

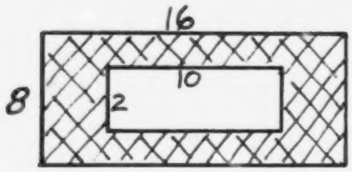
3. Develop an algorithm, formulate a plan, design an output and write the BASIC code for a program to compute an annual salary (AS) if a monthly salary (R) is entered.

SUMMARY:

What are the main components of a program to compute wages?

HOMEWORK:

1a. Find the the area of the outer rectangle.
 b. Find the area of the inner rectangle.
 c. Find the shaded area.



2. Compute the pay if the number of hours worked equals 32 and the hourly rate of pay equals $4.75.

3. Compute the commission if the rate of commission equals 14% and sales equal $2,245.

4. Revise Program I so that it will produce the following output:

```
PROGRAM TO DETERMINE WAGES
ENTER WEEKLY HOURS.
?20
ENTER HOURLY RATE OF PAY.
?5.25
THE WEEKLY WAGES ARE $105
```

5. Develop an algorithm, formulate a plan, design an output and write a program to determine an annual salary when a weekly salary is entered.

HOMEWORK SOLUTIONS:

1. a) 128 sq. units
   b) 20 sq. units
   c) 108 sq. units

2. $152 3. $314.30

4.
```
10 PRINT "PROGRAM TO DETERMINE WAGES"
20 PRINT "ENTER WEEKLY HOURS."
30 INPUT H
40 PRINT "ENTER HOURLY RATE OF PAY."
50 INPUT R
60 WS = H*R
70 PRINT "THE WEEKLY WAGES ARE $"; WS
80 END
```

5. Algorithm

YS=52·R where memory cell YS will store the annual salary and memory cell R will store the weekly rate of pay

Plan

1) Put in a value for the weekly rate of pay, R.
2) Calculate the yearly salary: YS=52·R.
3) Print YS.
4) End.

Planned Output
```
FINDING THE YEARLY SALARY
ENTER THE WEEKLY SALARY.
?450
$23400 IS EARNED EACH YEAR.
```

Program
```
10 PRINT "FINDING THE YEARLY SALARY"
20 PRINT "ENTER THE WEEKLY SALARY."
30 INPUT R
40 YS=52*R
50 PRINT "$";YS;"IS EARNED EACH YEAR."
60 END
```

Lesson: 24L

AIM: How do we code a program involving straight commission?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop an algorithm to determine a commission;
2. formulate a plan for a computer solution of the problem of determining commission;
3. design an output for a program to determine a commission;
4. write a program to determine a commission.

VOCABULARY: commission, rate of commission (commission rate)

MOTIVATION:

Beth earns a commission rate of 16% on sales of $18,000.
Brett earns a commission rate of 12% on sales of $22,000.
Who earns the higher commission?

DEVELOPMENT:

1. Ask the students to solve the motivation problem as shown below.
    a) Change each decimal to a percent; e.g., 16% = .16 and 12% = .12.
    b) Commission = Rate x Sales
       Beth (.16) x ($18,000) = $2,880
       Brett (.12) x ($22,000) = $2,640

   Therefore, Beth earns $240 more.
2. Ask, "How can we write a program to have the computer determine the amount of commission? What stages are involved in writing a program?"
   Elicit the steps that follow.
    a) Develop an algorithm
    b) Formulate a plan showing how to use the algorithm to solve the problem.
    c) Design an output.
    d) Code the program.
3. Present the following problem for a computer solution.

Problem: Enter the commission rate and amount of sales. Have the computer find and display the commission.

4. Ask, "*How do we* find the commission?"
   Elicit C = R·S where memory cell C stores the commission,
   memory cell R stores the commission rate,
   and memory cell S stores the sales.

5a. Ask, "How will we *use* the algorithm to solve the problem?"

Plan

1) Enter the values for the commission rate and sales: R,S.
2) Calculate the commission: C=R·S.
3) Print C.
4) End.

5b. Ask the students the following questions as you develop the plan.

i) What information must be entered into the computer?
ii) What algorithm do we want the computer to use in order to calculate the commission?
iii) What do we want the computer to display as the answer to our problem? In what memory cell is it located?

5c. Ask the students to develop a legend showing what each memory cell holds; e.g.,

C = commission
R = rate of commission
S = sales

6. Elicit a planned output from the students.

Planned Output

```
 COMPUTING A COMMISSION
 ENTER THE RATE (AS A DECIMAL) AND AMOUNT OF SALES.
?.16, 18000
 THE COMMISSION IS $2880
```

Point out the importance of the title, the instructions to the user and an explanation of the results. Ask, "How do we want the commission to be entered?" Elicit that the rate should be entered as a decimal number.

7a. Ask, "What are the BASIC statements that we will need in the program that will produce the output?"

Ask the students to code the program that follows and run it on the computer.

Program

```
10 PRINT "COMPUTING A COMMISSION"
20 PRINT "ENTER THE RATE (AS A DECIMAL) AND AMOUNT OF SALES."
30 INPUT R,S
40 C = R*S
50 PRINT "THE COMMISSION IS $"; C
60 END
```

(Call this "Program I".)

7b. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you.

7c. Circulate around the laboratory assisting students as needed.

7d. Group the students who are unable to write the program. Work through some of the coding with the students.

7e. Ask the students who complete the problem presented to solve the problems described in the Applications.

APPLICATIONS:

1. Revise the program written in today's lesson to output the following.

```
COMPUTING A COMMISSION
ENTER THE RATE AS A DECIMAL NUMERAL.
?.12
ENTER THE AMOUNT OF SALES.
?22000
THE COMMISSION = $2640
```

2. For the problem that follows, complete the instructions.
   a) Write an analysis that includes the development of an algorithm.
   b) Develop a plan that shows how to use the algorithm to solve the problem.

c) Design an output.

d) Code a program and RUN it on the computer.

Problem: Enter the rate of commission and sales for each of two salespeople. Have the computer find and display the total commission.

SUMMARY:

How do we have the computer determine the commission?

HOMEWORK:

1. Write 10111_2 as a decimal numeral.
2. Compute the commission if the amount of sales is $2,048 and the rate of commission is 9%.
3. Compare the annual salaries of Tom ($325 per week) and Carla ($1,410 per month).
4. Modify Program I so that it will produce the output below.

```
FINDING COMMISSION AMOUNTS
RATES ARE ENTERED IN DECIMAL FORM.
WHAT ARE THE RATE AND SALES AMOUNT
?.06,16500
COMMISSION AMOUNT = $990
```

5. Develop an algorithm, formulate a plan, design an output, and write a program to solve the problem.

Problem: Enter the hourly rate of pay, the number of hours, the commission rate and the sales. Have the computer determine the total earnings.

HOMEWORK SOLUTIONS:

1. 23

2. $184.32

3. Carla earns $16920 per year and Tom earns $16900 per year. Carla earns $20 more than Tom.

4.

```
10 PRINT "FINDING COMMISSION AMOUNTS"
20 PRINT "RATES ARE ENTERED IN DECIMAL FORM."
30 PRINT "WHAT ARE THE RATE AND SALES AMOUNT"
40 INPUT R,S
50 C=R*S
60 PRINT "COMMISSION AMOUNT = $"; C
70 END
```

5. Algorithm

$W = RP \cdot H$ where memory cell W stores the wages,
memory cell RP stores the hourly rate of pay,
and memory cell H stores the number of hours.

$C = R \cdot S$ where memory cell C stores the commission,
memory cell R stores the commission rate;
and memory cell S stores the sales.

$TE=W+C$ where memory cell TE stores the total earnings.

Plan

1) Put in values for the hourly rate of pay and the number of hours: RP,H.
2) Put in values for the commission rate and sales: R,S.
3) Calculate the wages: $W=RP \cdot H$.
4) Calculate the commission: $C=R \cdot S$.
5) Calculate the total earnings: $TE=W+C$.
6) Print TE.
7) End.

Planned Output

```
FINDING TOTAL EARNINGS
ENTER THE HOURLY RATE OF PAY.
?6
ENTER THE NUMBER OF HOURS WORKED.
?15
ENTER THE COMMISSION RATE IN DECIMAL FORM.
?.02
ENTER THE SALES.
?8500
EARNINGS ARE $260
```

Program

```
10  PRINT "FINDING TOTAL EARNINGS"
20  PRINT "ENTER THE HOURLY RATE OF PAY."
30  INPUT RP
40  PRINT "ENTER THE NUMBER OF HOURS WORKED."
50  INPUT H
60  PRINT "ENTER THE COMMISSION RATE IN DECIMAL FORM."
70  INPUT R
80  PRINT "ENTER THE SALES."
90  INPUT S
100 W = RP*H
110 C = R*S
120 TE = W+C
130 PRINT "EARNINGS ARE $"; TE
140 END
```

Lesson: 25

AIM: How do we determine the discount and the sale price?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the amount of discount;
2. find the percent of a number;
3. find the sale price;
4. demonstrate his/her understanding of finding the discount and the sale price by developing a plan for a computer solution.

VOCABULARY: discount, sale price, regular (original, list, marked) price, rate of discount

MOTIVATION:

Mr. Santos received 12% off the price of a computer that regularly sells for $450. How much did Mr. Santos pay for the computer?

DEVELOPMENT:

1a. While reviewing the motivation problem with the students, ask them to identify the regular price, the rate of discount, the discount and the sale price. Stress the difference between the rate of discount and the amount of discount.

1b. Ask the students to solve the motivation problem as follows.

i) Change 12% to a decimal. Remind the students that percent means "per hundred"; e.g., 12% means 12 per hundred or $\frac{12}{100}$. Since the bar means "divided by", to change a percent to a decimal, we must divide by 100. When we divide by 100, we move the decimal point two places to the left. For 12%, remove the percent sign: $12\cancel{\%}$

Locate the decimal point which is understood to be to the right of the 12: 12.

Move the decimal point two places to the left:
.12

ii) To determine the amount of discount, we must find .12 of $450 as shown below.
Discount = Rate of Discount x Regular Price
Discount = .12 x 450
Discount = $54
Mr. Santos saved $54 when he bought the computer on sale.

iii) To determine the sale price, we subtract the discount from the regular price as follows.
Sale Price = Regular Price - Amount of Discount
Sale Price = $450 - $54
Sale Price = $396

2. Ask the students to complete Application problems 1-5.

3. Present the problem that follows for computer solution.

Problem: Enter the regular price of an item and the rate of discount (expressed as a decimal). Have the computer find and display the amount of discount and the sale price.

4. Ask the students to develop the analysis that follows.
Analysis: Ask, "How do we find the amount of discount?"
Elicit D = RP·RD where memory cell D stores the discount, memory cell RP stores the regular price, and memory cell RD stores the discount, written as a decimal.
Ask, "How do we find the sale price?"
Elicit S = RP - D where memory cell S stores the sale price.

5a. Ask the students to develop the plan as follows. Remind the students to ask themselves, "How do we use the algorithm to solve the problem?"
Plan

1) Put in values for the regular price and the rate of discount (as a decimal): RP,RD.
2) Calculate the amount of discount: D = RP·RD.
3) Calculate the sale price: S = RP - D.

4) Display the amount of discount, D.
5) Display the sale price, S.
6) End.

5b. Ask the following questions as you guide the students in developing the plan.

  i) What values must we put into the computer? Tell the students that the regular price will be stored in memory cell RP and the rate of discount will be stored in memory cell RP.

  ii) What algorithm will the computer use to calculate the amount of discount?

  iii) What algorithm will the computer use to calculate the sale price?

  iv) What do we want the computer to display? In what memory cells are these values stored?

5c. Ask the students to formulate a legend showing the name of each memory cell and the data it will store; e.g.,

Memory cell	Stores
RP	Regular Price
RD	Rate of Discount
D	Amount of Discount
S	Sale Price

APPLICATIONS:

In 1-3, complete the table that follows by determining the discount and the sale price.

Item	Regular Price	Rate of Discount	Discount	Sale Price
1. Stereo	$415	20%		
2. Tape Recorder	$ 70	35%		
3. Color TV	$732	15%		

4. A merchant sells a $190 suit at a 10% reduction. What is the sale price?

5. The grocer you are working for tells you to reduce the price of an article 20%. What will be the new price if the article has been selling for 25¢?

SUMMARY:

1. Why do stores sell merchandise at a discount?
2. How do we determine the sale price of an item?
3. If the rate of discount is 15%, what percent of the regular price is the sale price?

HOMEWORK:

In 1-3, complete the table by determining the discount and the sale price.

Item	Regular Price	Rate of Discount	Discount	Sale Price
1. VCR	$325	10%		
2. AM/FM Pocket Radio	$ 15	25%		
3. Digital Watch	$ 40	8%		

4. How much will the discount be on a coat listed at $145 and sold at a 40% discount?
5. If a book listed at $19.50 is sold at a discount of 20%, for how much will the book sell?

HOMEWORK SOLUTIONS:

1. $32.50, $292.50
2. $3.75, $11.25
3. $3.20, $36.80
4. $58
5. $15.60

Lesson: 26L

AIM: How do we code a program to have the computer determine the sale price of an item?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to have the computer find the discount and sale price;
2. enter, run and correct his/her program.

VOCABULARY: discount, sale price, regular (original, list, marked) price, rate of discount

MOTIVATION:

Design a planned output and code a program that will solve the following problem.

Problem: Enter the regular price of an item and the rate of discount (expressed as a decimal). Have the computer find and display the amount of discount and the sale price.

DEVELOPMENT:

1a. Ask the students to create a planned output as follows. (Note to the teacher: Place on the chalkboard for those students who cannot design their own.)

Planned Output

```
FINDING DISCOUNTS AND SALE PRICES
PUT IN THE REGULAR PRICE AND
THE RATE OF DISCOUNT (IN DECIMAL FORM).
?450,.12
THE REGULAR PRICE IS 450 DOLLARS.
THE RATE OF DISCOUNT IS .12
THE AMOUNT OF DISCOUNT IS 54 DOLLARS.
THE SALE PRICE IS 396 DOLLARS.
```

1b. Point out the importance of the title, the instructions to the user, and an explanation of the results.

2a. Ask the students to code the program, enter it and run it.

Program

```
10  PRINT "FINDING DISCOUNTS AND SALE PRICES"
20  PRINT "PUT IN THE REGULAR PRICE AND"
30  PRINT "THE RATE OF DISCOUNT (IN DECIMAL FORM)."
40  INPUT RP,RD
50  D = RP*RD
60  S = RP - D
70  PRINT "THE REGULAR PRICE IS"; RP; "DOLLARS."
80  PRINT "THE RATE OF DISCOUNT IS"; RD
90  PRINT "THE AMOUNT OF DISCOUNT IS"; D; "DOLLARS."
100 PRINT "THE SALE PRICE IS"; S; "DOLLARS."
110 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

2b. Ask, "What effect would interchanging lines 70 and 80 with lines 50 and 60 have on the output?"

2c. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you.

2d. Ask those students who complete the program to solve the Application problem.

APPLICATION:

For the problem, corplete each of the following instructions.

1) Write an analysis of the problem that leads to an algorithm.
2) Write a plan that outlines how you use the algorithm to solve the problem.
3) Design a planned output.
4) Code a program and RUN it on the computer.

Problem: A store is having a sale offering 15% off on every item. Write a program to have the user enter the regular price of an item. Have the computer find and display the sale price.

SUMMARY:

The problems which follow are missing information that prevents their solution. What information is missing in each problem?

1. A storekeeper is having a sale in which she is offering 10% off every item. How much does Jose save when he purchases a jacket?
2. Tasha buys a sweater on sale that is usually listed at $35. How much does she pay for the sweater?

HOMEWORK:

For the problem, complete the following instructions.

a) Write an analysis of the problem that leads to an algorithm.
b) Write a plan that outlines how you use the algorithm to solve the problem.
c) Design a planned output.
d) Code a program.

Problem: If an item in a store is not selling, it is often discounted. If it still does not sell, it may be discounted again. Enter the regular price of the item, the first rate of discount, and the second rate of discount. Have the computer display the sale price after the first discount and then the sale price after the second discount.

HOMEWORK SOLUTIONS:

a) Analysis:

D1 = RP·R1 where memory cell D1 stores the first discount,
memory cell RP stores the regular price, and
memory cell R1 stores the first rate.

S1 = RP - D1 where memory cell S1 stores the first sale price.

D2 = S1·R2 where memory cell D2 stores the second discount and memory cell R2 stores the second rate of discount.

S2 = S1 - D2 where memory cell S2 stores the second sale price.

b) Plan

1) Put in value for the regular price, the first rate of discount, and the second rate of discount: RP,R1,R2.
2) Calculate the first discount: $D1 = RP \cdot R1$.
3) Calculate the first sale price: $S1 = RP - D1$.
4) Calculate the second discount: $D2 = S1 \cdot R2$.
5) Calculate the second sale price: $S2 = S1 - D2$.
6) Display the first sale price, S1.
7) Display the second sale price, S2.
8) End.

c) Planned Output

```
A DOUBLE DISCOUNT
ENTER THE REGULAR PRICE.
?100
ENTER THE FIRST RATE OF DISCOUNT.
?.1
ENTER THE SECOND RATE OF DISCOUNT.
?.2
THE FIRST SALE PRICE IS $90.
THE SECOND SALE PRICE IS $72.
```

d) Program

```
10  PRINT "A DOUBLE DISCOUNT"
20  PRINT "ENTER THE REGULAR PRICE."
30  INPUT RP
40  PRINT "ENTER THE FIRST RATE OF DISCOUNT."
50  INPUT R1
60  PRINT "ENTER THE SECOND RATE OF DISCOUNT."
70  INPUT R2
80  D1 = RP*R1
90  S1 = RP - D1
100 PRINT "THE FIRST SALE PRICE IS $"; S1;"."
110 D2 = S1*R2
120 S2 = S1 - D2
130 PRINT "THE SECOND SALE PRICE IS $"; S2;"."
140 END
```

Lesson: 27

AIM: How do we determine the final cost of an item including sales tax?

PERFORMANCE OBJECTIVES: The student will be able to...

1. compute the amount of sales tax on an item;
2. compute the final cost of an item.

VOCABULARY: sales tax rate

MOTIVATION:

Distribute copies of a newspaper advertisement for audio and video equipment; e.g., VCR $375 and blank tapes at $8.95 each. Ask, "What would be the total cost of one Video Cassette Recorder and 4 blank video tapes?"

DEVELOPMENT:

1. The students might respond to the motivation as follows.
   $375 + 4 x $8.95 = $375 + $35.80 = $410.80
2. Ask, "Why will we have to pay more than the $410.80?" Elicit that in New York City consumers must pay a sales tax on many goods.
   Ask, "If the sales tax rate is 8%, how could we find the cost of our purchases including sales tax?" Elicit that the amount of tax equals the tax rate multiplied by the price of the purchases. The final cost, including tax, would be the sum of the item cost and the sales tax. Say that for the video purchases, the sales tax is found by
   a) changing 8% to a decimal, 8% = .08,
   b) multiplying, .08 x $410.80 = $32.864 and
   c) rounding $32.864 to the nearest cent, $32.87
   (Sales tax is always rounded up.)
   Then the final cost is found as follows.
   $410.80 + $32.87 = $443.67
3. Ask the students to compute the sales tax and the final cost of other items that are advertised.

APPLICATIONS:

Compute the amount of sales tax and the final cost of the following items.

1. Cost: $45, tax rate: 8%
2. Cost: $21.55, tax rate: 8%
3. Cost: $140, tax rate: $8\frac{1}{4}$%
4. Cost: $195, tax rate: $8\frac{1}{2}$%
5. A stereo system costs $650 plus tax at a rate of 8%. Can we save money if the same stereo is available at $665 in an area where the sales tax rate is 4%?
6. Find the total cost before and after the sales tax for the following items.

Computer CPU	245.
Disc drive	290.
Color Monitor	195.
Printer	228.
Modem	99.

Use a tax rate of 8%.

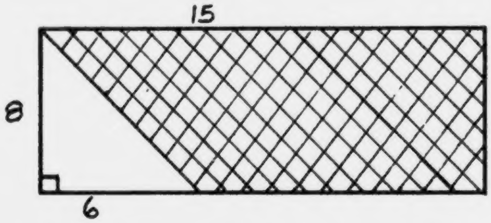
7. How much will be saved if the dealer in example 6 gives us a 15% discount before taxes?

SUMMARY:

What is the process for determining the final cost of an item including tax?

HOMEWORK:

1. a) Find the area of the rectangle.
   b) Find the area of the triangle.
   c) Find the area of the shaded region.



2. Compute the amount of pay for each of the following.
   a) 32 hours at $5.12 per hour
   b) 19% commission on sales of $4,200

3. Compute the amount of tax and final cost for the following items.
    - a) Price \$48, tax rate 5%
    - b) Price \$92, tax rate 8%
    - c) Price \$192, tax rate $8\frac{1}{4}$%
    - d) Price \$208, tax rate $8\frac{1}{4}$%

HOMEWORK SOLUTIONS:

1. a) 120 sq. units
   b) 24 sq. units
   c) 96 sq. units

2. a) \$163.84
   b) \$798

3. a) \$2.40, \$50.40
   b) \$7.36, \$99.36
   c) \$15.84, \$207.84
   d) \$17.16, \$225.16

Lesson: 28L

AIM: How do we code a program to determine the final cost of an item?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop an algorithm to determine the final cost of an item;
2. incorporate his/her algorithm into a plan;
3. design an output for a program to determine the final cost;
4. write a program to determine the final cost;
5. enter, run and correct the program.

VOCABULARY: rate of sales tax

MOTIVATION:

1. For the problem which follows, complete the instructions.
   a) Write an analysis that includes the development of an algorithm.
   b) Develop a plan that shows how to use the plan to solve the problem.
   c) Design an output.
   d) Code a program and run it on the computer.

> Problem: Enter the price of the item and the rate of sales tax. Have the computer find and display the amount of tax and the final cost of the item.

DEVELOPMENT:

1. Ask, "How do we find the amount of tax? What memory cells do we need? What is our algorithm? How do we find the final cost? What is the algorithm?"
   Elicit $T = R \cdot P$ and $C = P+T$ where
   memory cell T stores the amount of tax,
   memory cell R stores the rate of sales tax,
   memory cell P stores the price, and
   memory cell C stores the final cost.

2a. Ask, "How do we <u>use</u> the algorithm to solve the problem?"

2b. Ask the students to develop the plan that follows.

<u>Plan</u>

1) Put in the price and tax rate: P,R.
2) Calculate the tax: $T = R \cdot P$.
3) Calculate the final cost: $C = P+T$.
4) Print the amount of tax, T.
5) Print the final cost, C.
6) End.

2c. Elicit answers to the following questions as you develop the plan.

i) What information do we have to put into the computer? Remind the students that we will store the price in memory cell P and the tax rate in memory cell R.
ii) What algorithm do we want the computer to use in order to calculate the amount of tax?
iii) What algorithm do we want the computer to use to calculate the final cost?
iv) What do we want the computer to display?
v) What is the name of the memory cell where the amount of tax is stored? ...final cost is stored?

3a. Ask the students to plan an output design such as:

Planned Output

```
COMPUTING FINAL COST WITH SALES TAX
ENTER PRICE AND TAX RATE (AS A DECIMAL).
?128,.0825
AMOUNT OF TAX IS $10.56
FINAL COST IS $138.56
```

3b. Remind the students of the importance of the title, the instructions to the user and an explanation of the results.

4a. Ask the students to code the program that follows and run it on the computer.

Program

```
10 PRINT "COMPUTING FINAL COST WITH SALES TAX"
20 PRINT "ENTER PRICE AND TAX RATE (AS A DECIMAL)."
30 INPUT P, R
40 T = R*P
50 C = P + T
60 PRINT "AMOUNT OF TAX IS $"; T
70 PRINT "FINAL COST IS $"; C
80 END
```

4b. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you.

4c. Circulate around the laboratory assisting those students who need it.

4d. Group the students who are unable to write the program. Work through some of the coding with this group.

4e. Ask the students who complete the problem presented, to solve the problem described in the Application.

APPLICATION:

Complete the instructions for the problem which follows.

a) Write an analysis that leads to the development of an algorithm.
b) Write a plan that shows how you use the algorithm to solve the problem.
c) Design a planned output.
d) Code a program and RUN it on the computer.

Problem: Enter the price of an item. If the tax rate is 8%, have the computer find and display the sales tax on the item.

SUMMARY:

How do we determine the final cost of an item?

HOMEWORK:

Complete the instructions for the problem which follows.

a) Write an analysis that leads to the development of an algorithm.
b) Write a plan that shows how you use the algorithm to solve the problem.
c) Design a planned output.
d) Code a program.

Problem: All items in a store are being sold at a discount of 20% in a town where the sales tax rate is 8%. Enter the price of an item. Have the computer find the total cost of the item, including both the discount and the sales tax.

HOMEWORK SOLUTIONS:

a) Analysis

D = .2xP where memory cell D stores the discount and memory cell P stores the price of an item.

S = P-D where memory cell S stores the sale price.

T = .08xS where memory cell T stores the tax.

C = S+T where memory cell stores the final cost.

b) Plan

1) Put in a value for the price of the item, P.
2) Calculate the amount of discount; D = .2xP.
3) Calculate the sale price: S = P-D.
4) Calculate the amount of tax: T = .08xS.
5) Calculate the final cost: C = S+T.
6) Print C.
7) End.

c) Planned Output

```
THE FINAL COST OF AN ITEM
ENTER THE PRICE OF AN ITEM.
?30
THE ITEM COSTS $25.92
```

d) Program

```
10 PRINT "THE FINAL COST OF AN ITEM"
20 PRINT "ENTER THE PRICE OF AN ITEM."
30 INPUT P
40 D = .2*P
50 S = P-D
60 T = .08*S
70 C = S+T
80 PRINT "THE ITEM COSTS $"; C
90 END
```

Lesson: 29

AIM: What are the advantages of measuring length in the metric system?

PERFORMANCE OBJECTIVES: The student will be able to...

1. add and subtract linear measures in the English system;
2. add and subtract linear measures in the metric system;
3. state the advantages of using metric linear system measures.

VOCABULARY: metric system, English system

MOTIVATION:

Maria had 8 feet 4 inches of ribbon. If she used 2 feet 9 inches to trim a dress, how much ribbon was left?

DEVELOPMENT:

1. Elicit that in order to solve the motivation problem, we must subtract and in the process we must convert 1 foot into 12 inches; e.g.,

$$\begin{array}{rcrcr} \text{8 ft. 4 in.} & = & \text{7 ft. 12 + 4 in.} & = & \text{7 ft. 16 in.} \\ -\underline{\text{2 ft. 9 in.}} & = & -\underline{\text{2 ft. \quad 9 in.}} & = & -\underline{\text{2 ft. 9 in.}} \\ & & & & \text{5 ft. 7 in.} \end{array}$$

2. Ask, "What is incorrect about the following procedure?"

$$\begin{array}{rcr} \text{8 ft. 4 in.} & = & \text{7 ft. 14 in.} \\ -\underline{\text{2 ft. 9 in.}} & = & -\underline{\text{2 ft. 9 in.}} \\ & & \text{5 ft. 5 in.} \end{array}$$

   Elicit that the second method "exchanged 10" from the number of feet and added 10 to the number of inches. This is incorrect since there are 12, not 10, inches per foot. We encounter this situation in adding or subtracting inches, feet, yards, miles, ... Say to the students that because the metric system is decimal in nature, adding and subtracting different units is easier. Ask the students to compare the units in the tables that follow.

ENGLISH	METRIC
12 inches = 1 foot	1 millimeter (mm) = .1 centimeter (cm)
36 inches = 1 yard	1 centimeter = .1 decimeter (dm)
3 feet = 1 yard	1 decimeter = .1 meter (m)
1760 yards = 1 mile	1 meter = .1 dekameter (dkm)
5280 feet = 1 mile	1 dekameter = .1 hectometer (hm)
	1 hectometer = .1 kilometer (km)

3. The metric relationships that are used most often should be discussed with the students; e.g.,

   1 mm = .1 cm or 10 mm = 1 cm
   100 cm = 1 m
   1000 mm = 1 m
   1000 m = 1 km

   As a result, 8 cm 4 mm = 8.4 cm
   -2 cm 9 mm = 2.9 cm and so

$$\begin{array}{rcr} 8\text{ cm }4\text{ mm} & = & 8.4\text{ cm} \\ -\underline{2\text{ cm }9\text{ mm}} & = & -\underline{2.9\text{ cm}} \\ & & 5.5\text{ cm} \end{array}$$

4. Ask the students to solve and compare the problems below.

   Problem 1: Find the sum of 18 ft. 9 inches and 3 ft. 8 inches.

   Elicit that 9 inches + 8 inches = 17 inches = 1 ft. 5 inches.

   Therefore the sum is

$$\begin{array}{rl} 18\text{ ft.} & 9\text{ inches} \\ \underline{3\text{ ft.}} & \underline{8\text{ inches}} \\ 21\text{ ft.} & 17\text{ inches} = 21\text{ ft.} + 1\text{ ft. }5\text{ inches} \\ & = 22\text{ ft. }5\text{ inches.} \end{array}$$

   Problem 2: Find the sum of 18 m 9 dm and 3 m 8 dm.

   Elicit that the sum of 18 m 9 dm = 18.9 m
   and 3 m 8 dm = 3.8 m
   22.7 m = 22 m 7 dm.

APPLICATIONS:

1. Find the sum of...
   a) 3 ft. 6 in. + 5 ft. 8 in.
   b) 4 yd. 2 ft. 1 in. + 2 yd. 4 ft. 11 in.
   c) 8 cm 4 mm + 3 cm 7 mm
   d) 4 dm 2 cm 5 mm + 2 dm 9 cm + 8 mm

2. Find the perimeter of a rectangle whose length measures 4 cm 8 mm and whose width measures 5 cm 5 mm. Express your answer in centimeters.

SUMMARY:

1. Why is it easier to add and subtract different units of linear measure in the metric system?
2. Why do you think the use of the metric system is preferred in business and science?

HOMEWORK:

1. Find the total earnings of a worker earning wages plus commission if the number of hours worked is 35, the hourly rate is $6.50, the sales amount is $4,825 and the commission rate is 4%.
2. Find the final cost of an item priced at $14.95 with an 8% sales tax.
3. Find the sums.
    a) 12 ft. 9 inches + 8 ft. 6 inches
    b) 1 mile 3,050 ft. + 2 miles 3,920 ft.
    c) 1 kilometer 670 meters + 2 kilometers 743 meters
4. Find the differences.
    a) 6 ft. 2 inches - 3 ft. 4 inches
    b) 9 cm 2 mm - 4 cm 8 mm

HOMEWORK SOLUTIONS:

1. $420.50   2. $1.20

3. a) 21 ft. 3 in.   c) 4.413 km
   b) 4 miles 1690 ft.

4. a) 2 ft. 10 in.   b) 4.4 cm

Lesson: 30 L

AIM: How do we code a program to convert centimeters to millimeters?

PERFORMANCE OBJECTIVES: The student will be able to...

1. develop an algorithm to convert centimeters to millimeters;
2. incorporate the algorithm into a plan;
3. design an output for a program to convert centimeters to millimeters;
4. write the BASIC code for such a program.

VOCABULARY: no new terms

MOTIVATION:

How many millimeters are in a length of 8.7 centimeters?

DEVELOPMENT:

1. Ask, "How many millimeters are contained in each centimeter?" Elicit that since each centimeter contains 10 millimeters,

   8.7 centimeters = 8.7 x 1 centimeter
   = 8.7 x 10 millimeters
   = 87 millimeters.

2. Present the problem that follows for a computer solution.

   Problem: Enter the number of centimeters. Have the computer convert to millimeters.

3. Ask, "What would be needed in a computer program to convert centimeters to millimeters?"
   Elicit...
   a) an algorithm incorporated into a plan;
   b) an output design;
   c) a program code.

4. Ask, "How do we convert centimeters to millimeters? What

is our algorithm?" Elicit MM = 10·CM where memory cell MM stores the number of millimeters and memory cell CM stores the number of centimeters.

5a. Ask, "How do we use the algorithm to solve the problem?" Elicit the plan that follows.

Plan

1) Put in a value for the number of centimeters, CM.
2) Convert to millimeters: MM = 10·CM.
3) Display MM.
4) End

5b. As you develop the plan with your students, ask the following questions.

- i) What information do we need to enter into the computer?
- ii) What algorithm do we want the computer to use in order to make the conversion from centimeters to millimeters?
- iii) What value do we want the computer to display? In which memory cell is the value stored?

6. Elicit an output design such as the one that follows. Remind the students of the importance of the title, the instructions to the user and an explanation of the results.

Planned Output

```
CONVERTING CENTIMETERS TO MILLIMETERS
ENTER THE NUMBER OF CENTIMETERS.
?8.7
8.7 CM = 87 MM
```

7a. Ask the students to code the program that follows and run it on the computer.

Program

```
10 PRINT "CONVERTING CENTIMETERS TO MILLIMETERS"
20 PRINT "ENTER THE NUMBER OF CENTIMETERS."
30 INPUT CM
40 MM=10*CM
50 PRINT CM; "CM ="; MM; "MM"
60 END
```

The spacing will vary depending on the computer you are using. Vary the program accordingly.

7b. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

7c. Circulate around the laboratory assisting students as needed.

7d. Group the students who are unable to write the program. Work through some of the coding with the students.

7e. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

1. Modify the program written in this lesson so that it will produce the planned output below.

```
HOW MANY MILLIMETERS?
TYPE THE NUMBER OF CENTIMETERS.
? 324
THE NUMBER OF MILLIMETERS IS 3240
```

2. For the problem that follows, complete the instructions.
   a) Write an algorithm.
   b) Develop a plan showing how you use the algorithm to solve the problem.
   c) Design an output.
   d) Code a program and RUN it on the computer.

Problem: Enter the number of kilometers. Have the computer convert to meters.

SUMMARY:

What are the essentials of a program to convert centimeters to millimeters?

HOMEWORK:

1. A rectangular room is 8 meters long and 3.5 meters wide. Find the cost of carpeting the room if the carpet costs $12 per square meter.

2. A small jewelry box is 8 dm 4 cm long and 6 dm 8 cm wide. A thin line of gold paint is painted around the boundary at the top of the box. How long is the perimeter,

expressed in terms of cm? If the paint costs $.10 per cm, find the cost of the paint.

3. For the problem that follows, complete the instructions.
   a) Write an algorithm.
   b) Develop a plan showing how you use the algorithm to solve the problem.
   c) Design an output.
   d) Code a program.

Problem: Enter the number of meters. Have the computer convert to centimeters.

HOMEWORK SOLUTIONS:

1. $336
2. 304 cm, $30.40

3. a) Algorithm

   CM = 100·M where memory cell CM stores the number of centimeters and memory cell M stores the number of meters.

   b) Plan
      1) Put in the number of meters, M.
      2) Convert to centimeters: CM = 100·M.
      3) Display CM.
      4) End.

   c) Planned Output

```
CHANGING METERS TO CENTIMETERS
ENTER THE NUMBER OF METERS.
?6
# OF CM       # OF M
600           6
```

   d) Program

```
10 PRINT "CHANGING METERS TO CENTIMETERS"
20 PRINT "ENTER THE NUMBER OF METERS."
30 INPUT M
40 CM = 100*M
50 PRINT "# OF CM","# OF M"
60 PRINT CM, M
70 END
```

Lesson: 31

AIM: What are supplementary and complementary angles?

PERFORMANCE OBJECTIVES: The student will be able to...

1. state the definitions of supplementary and complementary angles;
2. determine the measure of the supplement of a given angle;
3. determine the measure of the complement of a given angle;
4. identify two adjacent supplementary or complementary angles in a diagram;
5. sketch a diagram of supplementary or complementary angles and use it to to solve a problem.

VOCABULARY: angle, vertex, side, adjacent angles, measure of an angle, supplementary angles, complementary angles, perpendicular, coplanar, interior

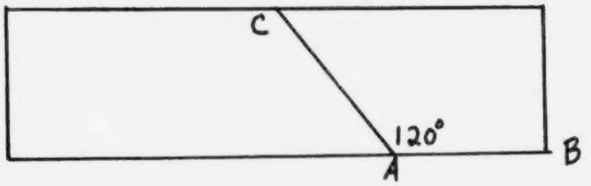
SYMBOLS: ⊥, ⟶, < or ∡, °(degree)

MOTIVATION:

A 2x6 board must be cut at an angle of 120° to fit as a roof beam. The saw can be adjusted to make cuts between 0° and 90°. To what angle measure should the saw be set?

DEVELOPMENT: (Note: In this lesson, the applications are contained within the development.)

1a. Refer to motivation. Elicit a diagram for the problem.



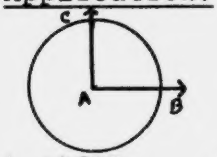
As needed, ask the students to review the definitions and notation as follows.

i) An angle is the union of two rays (sides) with a common endpoint (vertex). (Noted <BAC or <A.)

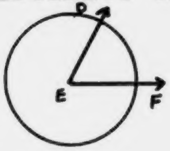
ii) A degree is $\frac{1}{360}$ of a complete circular rotation of one side of an angle. (Noted 1°.)

iii) The measure of an angle is the size of the interior of an angle determined by comparison to a unit measure such as a degree. (Noted m∡BAC = 60.)

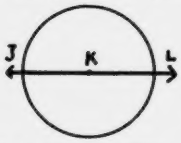
1b. Application: Estimate the measure of each angle.



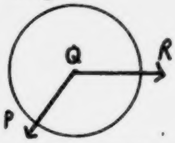
i) m∡CAB=



ii) m∡DEF=



iii) m∡JKL=

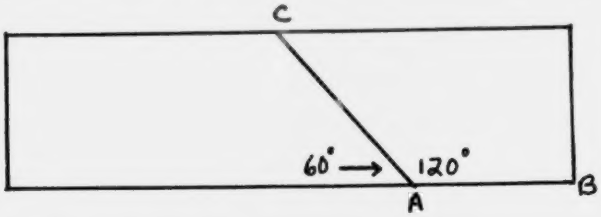


iv) m∡PQR=

What measure is formed by these rotations?

v) $\frac{1}{3}$ circle vi) $\frac{1}{4}$ circle vii) $\frac{1}{8}$ circle viii) $\frac{5}{6}$ circle

1c. Elicit a solution to the motivation problem.



2a. Define supplementary angles; e.g.,
Supplementary angles are two angles whose measurements total 180°.

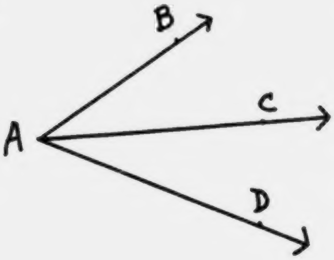
2b. Application 2: Determine the measure of the supplement of each angle whose measurement is listed.

i) 125° ii) 50° iii) 90° iv) $162\frac{1}{2}$° v) 78.4°

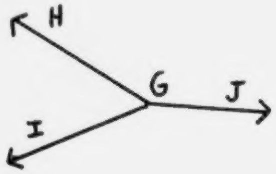
3a. Define adjacent angles; e.g.,
Adjacent angles are two coplaner angles which have a common vertex and common side but no common interior points.

3b. Ask, "In which diagram do you find a pair of supplementary adjacent angles?"

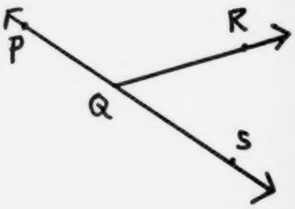
i)



ii)



iii)



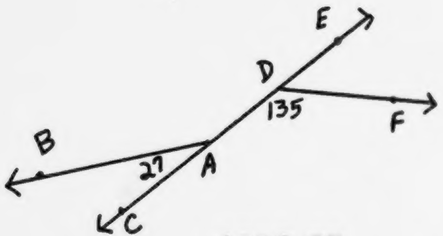
3c. Ask, "Which are the sides of ∡PQR? ...∡RQS? Which side do ∡PQR and ∡RQS have in common? Which are the noncommon sides? What geometric figure is formed by the noncommon sides?"

Elicit: Two adjacent angles are supplementary when their noncommon sides form a straight line.

3d. Application 3:

i) Name pairs of supplementary angles.

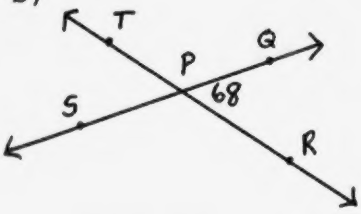
a)



m∡BAC=27

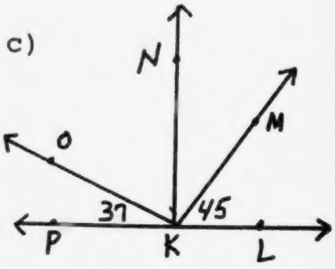
m∡FDC=135

b)



m∡QPR=68

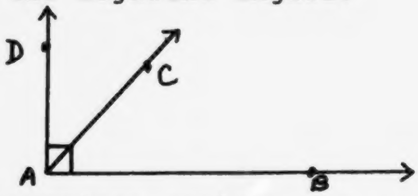
c)



m∡PKO=37
m∡LKN=90
m∡MKL=45

ii) Determine the measure of each member of the supplementary angle pairs.

4a. Ask, "In the following diagram, what is the sum of the measures of the adjacent angles?"



4b. Define complementary angles; e.g.,

Complementary angles are two angles whose measurements total 90°.

4c. Application 4: Determine the measure of the complement of each angle whose measurement is listed.

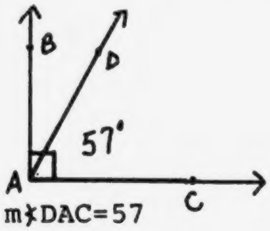
i) 53°   ii) 45°   iii) $16\frac{3}{4}$°   iv) 83.25

4d. Referring to the diagram in 4a, ask, "Which are the sides of ∡DAC? ...∡CAB? Which side do ∡DAC and ∡CAB have in common? Which are the noncommon sides? What do we call two rays that meet and form a right angle?" (Review the notation; e.g., $\overrightarrow{AD} \perp \overrightarrow{AB}$ means ray AD is perpendicular to ray AB.

Elicit: Two adjacent angles whose noncommon sides form a right angle (are perpendicular) are complementary.

4e. Application 5: Name the complementary angles and determine their measures.

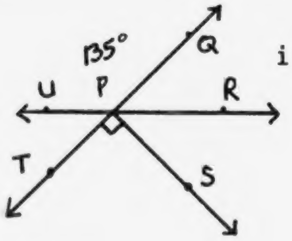
i)



$m\measuredangle DAC=57$

$\overrightarrow{AB}\perp\overrightarrow{AC}$

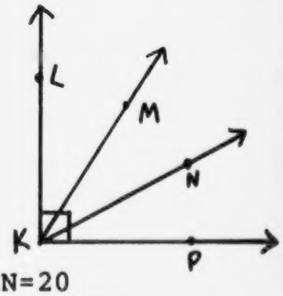
ii)



$m\measuredangle UPQ=135$

$\overrightarrow{PT}\perp\overrightarrow{PS}$

iii)



$m\measuredangle MKN=20$
$m\measuredangle MKL=m\measuredangle NKP$

$\overrightarrow{KL}\perp\overrightarrow{KP}$

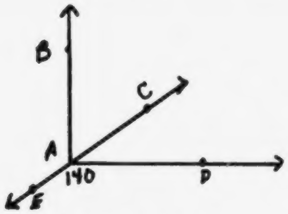
SUMMARY:

1. What should we look for to find a pair of supplementary angles in a diagram? ...a pair of complementary angles?
2. What algorithm can be used to determine the supplement of a given angle? ...the complement of a given angle?

HOMEWORK:

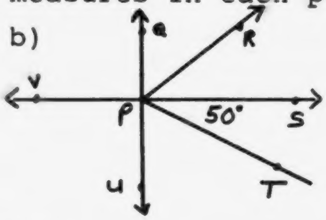
1. Determine the measurements of angles formed by the following rotations.
   a) $\frac{1}{2}$ circle b) $\frac{3}{4}$ circle c) $\frac{5}{8}$ circle d) $\frac{2}{3}$ circle
2. Determine the measure of the supplement of each angle whose measure is listed.
   a) 85 b) 143 c) 92.5
3. Determine the measure of the complement of each angle whose measure is listed.
   a) 47 b) 82.5 c) 67.125
4. Name pairs of supplementary and complementary angles in each diagram and determine the measures in each pair.

a)



$m\measuredangle DAE=140$
$m\measuredangle BAD=90$, $\overleftrightarrow{EC}$

b)



$\overleftrightarrow{VS}$ $\overleftrightarrow{QU}$, $m\measuredangle RPT=90$,
$m\measuredangle SPT=50$

5. Determine the measurements of two supplementary angles such that one is twice the other.

HOMEWORK SOLUTIONS:

1. a) 180° c) 225°
   b) 270° d) 240°

2. a) 95 c) 87.5
   b) 37

3. a) 43 c) 22.875
   b) 7.5

4. a) ∡BAE is supplementary to ∡BAC.
   ∡DAE is supplementary to ∡DAC.
   ∡BAC is complementary to ∡DAC.
   m∡DAC = 40, m∡BAC = 50, m∡BAE = 130

   b) ∡VPR is supplementary to ∡RPS.
   ∡VPT is supplementary to ∡SPT.
   ∡RPS is complementary to ∡SPT.
   ∡QPS is complementary to ∡RPS.
   ∡SPT is complementary to ∡TPU.
   ∡VPQ is supplementary to ∡QPS.
   ∡VPU is supplementary to ∡UPS.
   ∡VPQ is supplementary to ∡VPU.
   ∡QPS is supplementary to ∡SPU.
   ∡QPT is supplementary tp ∡TPU.
   ∡RPQ is supplementary to ∡RPU.
   m∡VPT = 130, m∡RPS = 40, m∡VPR = 140, m∡QPS = 50,
   m∡UPT = 40, m∡QPT = 140, m∡RPU = 130, m∡VPQ = 90,
   m∡QPS = 90, m∡VPU = 90, m∡UPS = 90

5. 120°, 60°

Lesson: 32L

AIM: How do you write a program to determine the measure of the supplement of an angle?

PERFORMANCE OBJECTIVES: The student will be able to...

1. determine and use an algorithm for finding the supplement of a given angle;
2. develop a plan which incorporates the algorithm for finding the supplement;
3. design an output;
4. write a program from the plan and planned output;
5. trace the program for a set of given measures;
6. enter, run and debug the program for finding the supplement.

VOCABULARY: complement, supplement, acute

MOTIVATION:

Determine the measure of the supplement of each angle whose measure is listed.

a) 132.5 b) 89 c) 54

DEVELOPMENT:

Ask, "What are steps in preparing a problem for computer solution?"

1. Analysis:
   a) Ask, "If A represents the measure of an angle, what algorithm would produce the supplement?"
   Supplement = 180 - A
   b) Ask, "How do we use this algorithm to find the supplement of an angle whose measure is 89?"
   S = 180 - A
   S = 180 - 89
   S = 91
2. Ask, "How can this algorithm be used in a program to find the measure of the supplement of an angle?"

Elicit the following plan by asking the students to write the steps they followed to use the algorithm, S = 180-A, to find the supplement of an angle whose measure is stored in memory cell A.

<u>Plan</u>:

1) Put in a value for the measure of an angle, A.
2) Calculate the supplement: S = 180 - A.
3) Print S.
4) End.

3. Ask the students to develop a planned output. Remind them to include a title, a prompt and input (instructions to the user), and an (explanation of the) answer.

		Planned Output
TITLE	↔	CALCULATING THE SUPPLEMENT
PROMPT	↔	ENTER THE MEASURE OF AN ANGLE.
INPUT	↔	?62.7
ANSWER	↔	THE SUPPLEMENT MEASURES 117.3

4a. Ask the students to write the program.

Program

```
10 PRINT "CALCULATING THE SUPPLEMENT"
20 PRINT "ENTER THE MEASURE OF AN ANGLE."
30 INPUT A
40 S = 180 - A
50 PRINT "THE SUPPLEMENT MEASURES";A
60 END
```

4b. Ask the students to trace the program for values of A and construct a chart as follows.

A	S
12	168

4c. Ask the students to enter the program into the computer and check its accuracy using the previously calculated values.

If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

4d. Circulate around the laboratory assisting students as needed.

4e. Group the students who are unable to write the program. Work through some of the coding with the students.

5. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

1. Code the program which will produce the following variations in the output.

Planned Output

```
TO FIND THE SUPPLEMENT
ENTER AN ANGLE MEASURE.
?18
THE SUPPLEMENT OF 18 DEGREES MEASURES 162 DEGREES.
```

2. Complete the instructions for the problem that follows.
   a) Write an analysis which includes the development of an algorithm.
   b) Develop a plan which shows how you use the algorithm to solve the problem.
   c) Design an output.
   d) Code a program and RUN it on the computer.

<u>Problem</u>: Enter the measure of an angle (between 0 and 90). Have the computer find and display the measure of its complement.

SUMMARY:

What information does the computer need in order to determine the measure of the supplement of an angle? ...the complement of an angle?

HOMEWORK:

1. Determine the measures of the supplement and the complement of each angle whose measurement is listed below.

   a) 65 b) 12.5 c) 81.75

2. Complete the instructions for the problem that follows.
   a) Write an analysis which includes the development of an algorithm.
   b) Develop a plan which shows how you use the algorithm to solve the problem.

c) Design an output.

d) Code a program.

> Problem: Enter the measure of an angle (between 0 and 90). Have the computer find and display the measures of the supplement and complement of an angle.

HOMEWORK SOLUTIONS:

1. a) 115, 25 c) 98.25, 8.25
   b) 167.5, 77.5

2a) Analysis

S=180-A where memory cell S stores the measure of the supplement and memory cell A stores the measure of the angle.

C=90-A where memory cell C stores the measure of the complement.

2b) Plan

1) Put in the measure of an angle, A.
2) Calculate the measure of the supplement: S=180-A.
3) Calculate the measure of the complement: C=90-A.
4) Display S and C.
5) End.

2c) Planned Ouput

```
FINDING THE SUPPLEMENT AND COMPLEMENT
ENTER THE MEASURE OF AN ACUTE ANGLE.
?35
THE SUPPLEMENT MEASURES 145 DEGREES.
THE COMPLEMENT MEASURES 55 DEGREES.
```

2d) Program

```
10 PRINT "FINDING THE SUPPLEMENT AND COMPLEMENT"
20 PRINT "ENTER THE MEASURE OF AN ACUTE ANGLE."
30 INPUT A
40 S = 180 - A
50 C = 90 - A
60 PRINT "THE SUPPLEMENT MEASURES";S;"DEGREES."
70 PRINT "THE COMPLEMENT MEASURES";C;"DEGREES."
80 END
```

Lesson: 33

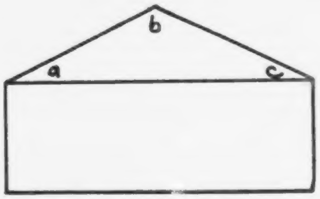
AIM: What is the sum of the interior angles of a triangle?

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate that the sum of the measures of the interior angles of a triangle is 180;
2. determine the measure of an interior angle of a triangle when given the measures of two angles.

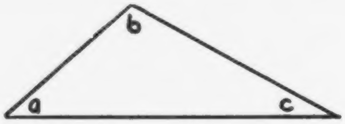
MOTIVATION:

James is building a dog house. The roof will be made to come to a point at an angle measuring 80°. To plan ahead, he must figure out the angles at the base. One "base" angle is to measure 60°. What is the measure of the other?

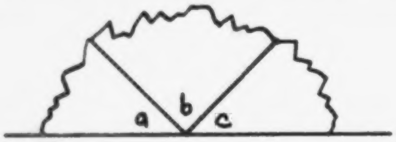


DEVELOPMENT:

1. To elicit that the sum of the measures of the interior angles is 180, do the following.
   a) Distribute various kinds of paper triangles. Ask the students to label the interior angles a, b and c.



   b) Ask, "If we cut off and put together angles a, b and c as follows, what type of angle is formed?" Demonstrate at the chalkboard.



   c) Ask the students to do the same for the distributed triangles.

   d) Ask, "What is the sum of the interior angles of a triangle?"

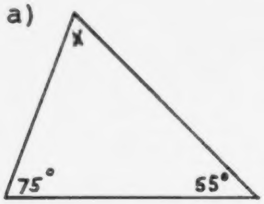
2. Refer back to the motivation for solution; e.g.,

m∠A = 180 - (60 + 80)
m∠A = 180 - 140
m∠A = 40

APPLICATIONS:

1. Find the value of x in each.

a)



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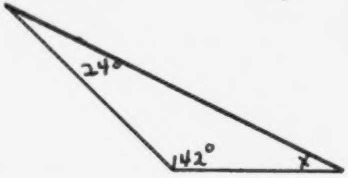
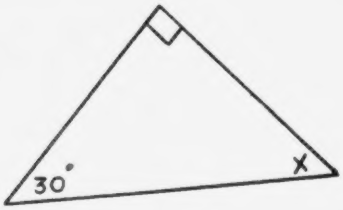


c)



2. Draw a triangle, label it, and then find the measure of each angle.
   - a) m∠A = 35; m∠B = 49; m∠C = ?
   - b) m∠A = 45; m∠B = 90; m∠C = ?
   - c) m∠D = 132; m∠E = 27; m∠F = ?
   - d) The m∠A = 80 and the measure of the other two angles, ∠B and ∠C, are equal.
   - e) The measure of all the angles, ∠R, ∠S and ∠T, are equal.
   - f) The first angle, ∠X, is a right angle and the other two, ∠Y and ∠Z, are equal in measure.
   - g) The first angle, ∠G, is a right angle and the second, ∠H, measures twice the third, ∠I.

SUMMARY:

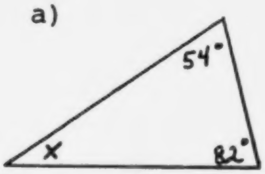
1. What do we know about the sum of the interior angles of any triangle?

2. If we are given the measure of two angles of a triangle, how do we find the measure of the third angle?

HOMEWORK:

1. Determine the value of x in each.

a)



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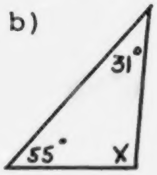
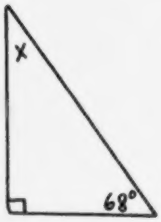


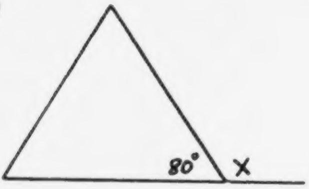
c)



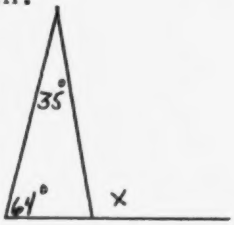
2. Draw a diagram, label and solve.
   a) In a triangle each of two angles measures 70°. What is the measure of the third angle?
   b) In a triangle each angle has the same measure. Is the measure equal to 50? Explain your answer.
   c) Can a triangle consist of two right angles? Explain.
   d) Can two angles of a triangle be supplementary? Explain using examples.
   e) Can two angles of a triangle be complementary? Explain using examples.

3. Determine the value of x in each.

a)



b)



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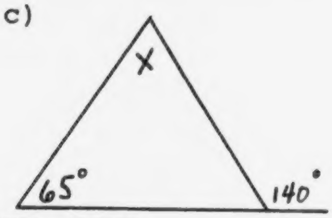

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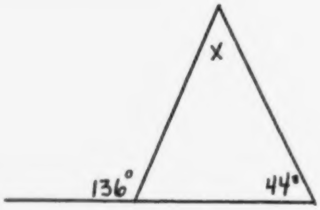
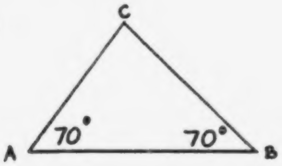


HOMEWORK SOLUTIONS:

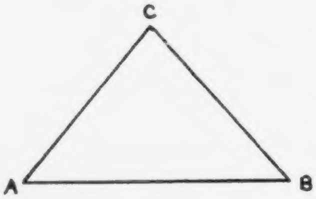
1. a) x = 44
   b) x = 94
   c) x = 22

2. a)



m∡C = 40

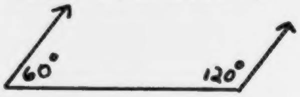
b)



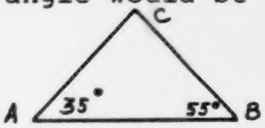
The measure cannot be 50 because 3 x 50 ≠ 180. The measure is 60.

c) A triangle cannot consist of two right angles because the third angle would not exist; e.g.,

d) Two angles of a triangle cannot be supplementary. The third angle would not exist; e.g.,



e) The angles of a triangle can be complementary. The third angle would be 90; e.g.,



3. a) x = 100° c) x = 75°
b) x = 99° d) x = 92°

(This question will be used in lesson 34L to establish the relationship that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two interior angles that are not adjacent to it. (See applications 2, Lesson 34L.)

Lesson 34L

AIM: How do we code a program to have the computer find the measure of the third angle of a triangle when given the measures of two of the angles?

PERFORMANCE OBJECTIVES: The student will be able to...

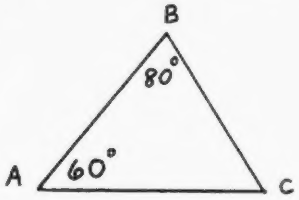
1. determine and use an algorithm for finding the measure of the third angle of a triangle;
2. develop a plan;
3. design an output;
4. write a program using both the plan and planned output;
5. enter, trace and debug the program.

VOCABULARY: no new terms

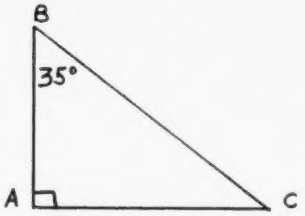
MOTIVATION:

Given triangle ABC and the measures of angles A and B, find the measure of angle C in each.

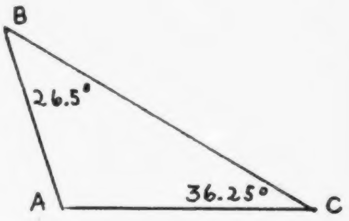
a)



b)



c)



DEVELOPMENT:

1. After the students have solved the problems above, say, "Let us find a computer solution to the above problems." Ask, "How can we state the problem?"

   Problem: Enter the measures of two angles of a triangle. Have the computer find and display the measure of the third angle of the triangle.

2. Develop the analysis that follows with the students.

   Analysis

   a) Ask, "How can we represent the measure of the angles?"

Elicit: Let MA = the measure of angle A.
Let MB = the measure of angle B.
Let MC = the measure of angle C.

b) Ask, "What algorithm can be used to determine the value of MC when given values for MA and MB?" Refer to the motivation problems to elicit MC = 180 - (MA + MB). Stress the need for the parentheses.

3. Ask, "How can this algorithm be incorporated into a plan? How do we <u>use</u> it to solve the problem? What information must the computer be given? What memory cell will hold the solution to the problem?"

   Elicit the plan below.

   <u>Plan</u>

   1) Put in values for the measures of the two angles: MA, MB.
   2) Calculate the measure of the third angle:
      MC = 180-(MA + MB).
   3) Print MC.
   4) End.

4. Ask, "What do we want to see on the screen when we run our program? What information is necessary to explain what our program does? ...what the user should enter? ...the results?"

   <u>Planned Output</u>

```
TO FIND THE MEASURE OF ANGLE C
WHAT IS THE MEASURE OF ANGLE A AND ANGLE B?
?75,25
ANGLE C MEASURES 80 DEGREES.
```

5. <u>Program</u>

   Ask the students to write the program.

```
10 PRINT "TO FIND THE MEASURE OF ANGLE C"
20 PRINT "WHAT IS THE MEASURE OF ANGLE A AND ANGLE B?"
30 INPUT MA,MB
40 MC = 180 - (MA + MB)
50 PRINT "ANGLE C MEASURES"; MC; "DEGREES."
60 END
```

The spacing varies depending on the computer used. Vary the program accordingly.

6. Ask, "How do we test the program to see if it works before we go to the computer?" Ask the students to trace the program with different values of MA and MB. Ask them to list their data in chart form.

MA	MB	MC
30	40	110
125	35	20

Ask the students to select three "test" values to trace their program. Suggest values other than those of whole numbers.

7a. Ask the students to run their programs. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you to grade.

7b. Circulate around the laboratory assisting students as needed.

7c. Group the students who are unable to write the program. Work through some of the coding with this group.

7d. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

1. Change your program to match the following planned output.

Planned Output

```
TO FIND THE MEASURE OF ANGLE C
WHAT IS THE MEASURE OF ANGLE A AND ANGLE B?
?75,25
M<A=75
M<B=25
M<C=80
```

2. Complete the instructions for the problem that follows.
   a) Write an analysis that includes the development of an algorithm.
   b) Formulate a plan that shows how you use the algorithm to solve the problem.
   c) Design a planned output.
   d) Code a program and RUN it on the computer.

Problem: Enter the measures of two interior angles of a triangle. Have the computer find and display the measure of the exterior angle at the third angle.
(Note: To assist students in solving the problem, refer them to the homework from Lesson 33.)

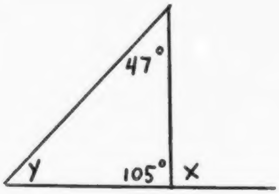
SUMMARY:

How does the program for finding the measure of the third angle of a triangle differ from the one for finding the supplement of a given angle?

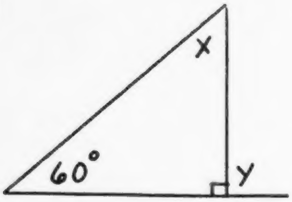
HOMEWORK:

1. Determine the values of x and y.

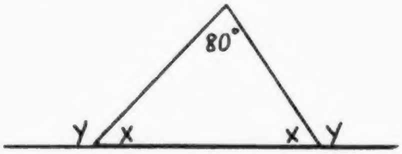
a)



b)



c)



2. Complete the instructions for the problem and planned output which follow.
   - a) Write an analysis that includes the development of an algorithm.
   - b) Formulate a plan that shows how you use the algorithm to solve the problem.
   - c) Code a program that will produce the given planned output.

Problem: Enter the measures of two interior angles of a triangle. Have the computer find and display the measure of the exterior angle formed at the third angle. Then have it find and display the measure of the third interior angle.

Planned Output

```
FINDING THE EXTERIOR AND INTERIOR ANGLES OF A TRIANGLE
ENTER THE MEASURES OF TWO INTERIOR ANGLES.
?35,85
EXTERIOR ANGLE C MEASURES 120 DEGREES.
INTERIOR ANGLE C MEASURES 60 DEGREES.
```

HOMEWORK SOLUTIONS:

1. a) $x = 75$, $y = 28$ c) $x = 50$, $y = 130$
b) $x = 30$, $y = 90$

2a) Analysis

XC = IA+IB where
memory cell XC holds the value of the exterior angle C,
memory cell IA holds the value of interior angle A, and
memory cell IB holds the value of interior angle B.
IC = 180-XC where memory cell IC holds the value of interior angle C.

2b) Plan

1) Enter the measures of two interior angles of a triangle: IA,IB.
2) Calculate the measure of the exterior angle: XC = IA + IB.
3) Calculate the measure of the interior angle: IC = 180 - XC.
4) Print XC, IC.
5) End.

2c) Program

```
10 PRINT "FINDING THE EXTERIOR AND INTERIOR ANGLES OF A TRIANGLE"
20 PRINT "ENTER THE MEASURES OF TWO INTERIOR ANGLES."
30 INPUT IA, IB
40 XC = IA + IB
50 IC = 180 - XC
60 PRINT "EXTERIOR ANGLE C MEASURES"; XC;"DEGREES."
70 PRINT "INTERIOR ANGLE C MEASURES"; IC;"DEGREES."
80 END
```

The spacing will vary depending on the computer used. Ask the students to adjust their programs accordingly.

Lesson: 35

AIM: How do we find the measure of the fourth angle of a quadrilateral when given the measures of the other three angles?

(The material in this lesson can be developed in one full period. To actually have the students run the program will require a session in the computer laboratory.)

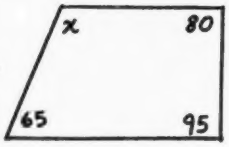
PERFORMANCE OBJECTIVES: The student will be able to...

1. state and demonstrate that the sum of the measures of the angles of a quadrilateral is 360;
2. determine the measures of the interior angles of a quadrilateral;
3. analyze and develop a plan for a computer solution for finding the measure of the fourth angle of a quadrilateral, given the measures of the other three angles;
4. design a planned output;
5. code, trace, enter and debug his/her program.

VOCABULARY: polygon, quadrilateral, diagonal, pentagon, hexagon, octagon, septagon

MOTIVATION:

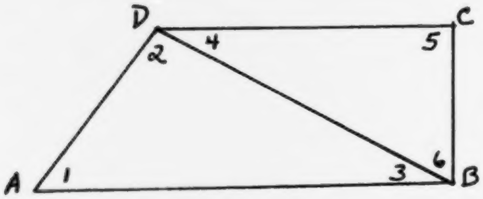
Find the value of x.



DEVELOPMENT:

1. Refer to motivation.
   a) Ask, "What do we need to know in order to find x?" (The sum of the measures of the interior angles of a quadrilateral.)

b) Elicit that the sum is 360 by drawing a diagonal to divide a quadrilateral into two triangles.



i) $m\angle 1+m\angle 2+m\angle 3 = 180$, $m\angle 4+m\angle 5+m\angle 6 = 180$

ii) $m\angle 1+m\angle 2+m\angle 3+m\angle 4+m\angle 5+m\angle 6 = 360$

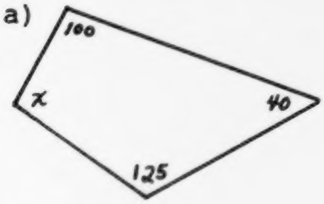
iii) Since $m\angle 2+m\angle 4 = m\angle D$ and $m\angle 3+m\angle 6 = m\angle B$, $m\angle A+m\angle B+m\angle C+m\angle D = 360$.

This may also be elicited by using specific degree measures.

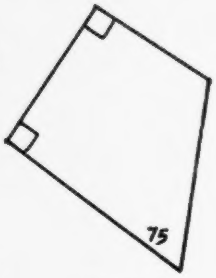
c) Ask the students to solve the motivation problem.

2. Determine the value of x.

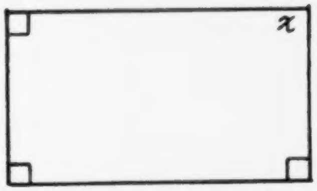
a)



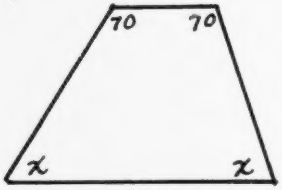
b)



c)



d)



3. Ask, "How do we find the measure of the fourth angle of a quadrilateral?" Elicit the algorithm for finding the measure of the fourth angle, given three angles of a quadrilateral...MD = 360-(MA+MB+MC) where MD stores the measure of the fourth angle and MA, MB and MC store the measures of the three given angles.

4a. Assign to the students the task of developing a plan for a program to find the measure of the fourth angle of a quadrilateral.

4b. Remind the students to ask themselves, "How do we use the algorithm to solve the problem?"

4c. Form groups to work on the plan. Circulate around the classroom assisting students as needed. Ask the following questions as you assist the students.

  i) What information do we need to enter into the computer? What are the names of the memory cells that will store that data?
  ii) What algorithm will the computer use to calculate the measure of the fourth angle?
  iii) What do we want the computer to display? In what memory cell is this information stored?

4d. Elicit the plan below from each group.

<u>Plan</u>

1) Put in values for the measures of three angles of a quadrilateral: MA,MB,MC.
2) Calculate the measure of the fourth angle: MD = 360 - (MA + MB + MC).
3) Print MD.
4) End.

5. <u>Planned Output</u>: (You may ask the students to develop their own.)

Ask the students to code a program that will produce the planned output that follows. Ask the students to notice the elements of a planned output... the title, the instructions to the user and an explanation of the results.

<u>Planned output</u>

```
FINDING THE FOURTH ANGLE OF A QUADRILATERAL
WHAT ARE THE MEASURES OF THREE ANGLES?
?80,120,50
THE FOURTH ANGLE MEASURES 110 DEGREES.
```

6. Assign the students to code the program from their plan and planned output. (Circulate and help the students.)

<u>Program</u>

```
10 PRINT "FINDING THE FOURTH ANGLE OF A QUADRILATERAL"
20 PRINT "WHAT ARE THE MEASURES OF THREE ANGLES?"
30 INPUT MA, MB, MC
40 MD=360-(MA+MB+MC)
50 PRINT "THE FOURTH ANGLE MEASURES";MD;"DEGREES."
60 END
```

APPLICATIONS:

1. Trace your program for the following values and fill in column MD.

MA	MB	MC	MD
82	63	58	
90	90	40	
90	90	90	
62.5	83.25	142.25	

2. What is the sum of the measures of the interior angles of a pentagon?
3. What algorithm would be used to find the measure of the fifth angle of a pentagon?
4. What is the sum of the measures of the interior angles of a hexagon? ...septagon?...octagon?

SUMMARY:

If we are given the measures of three angles of a quadrilateral, how do we find the measure of the fourth angle?

HOMEWORK:

Complete the instructions for the problem that follows.

a) Write an analysis that includes the development of an algorithm.
b) Develop an algorithm that shows how to use your algorithm to solve the problem.
c) Design a planned output.
d) Code a program.

Problem: Enter the measures of four interior angles of a pentagon. Have the computer find and display the measure of the fifth angle.

HOMEWORK SOLUTIONS:

a) Analysis

ME = 540 - (MA+MB+MC+MD)

where memory cell ME stores the measure of the fifth angle and memory cells MA, MB, MC and MD store the measures of the four given angles.

b) Plan

1) Put in values for the measures of four angles of the pentagon: MA,MB,MC,MD.
2) Calculate the measure of the fifth angle: ME = 540 - (MA+MB+MC+MD).
3) Print ME.
4) End.

c) Planned Output

```
FINDING THE FIFTH ANGLE OF A PENTAGON
WHAT ARE THE MEASURES OF FOUR ANGLES?
?110,90,40,210
THE FIFTH ANGLE MEASURES 90 DEGREES.
```

d) Program

```
10 PRINT "FINDING THE FIFTH ANGLE OF A PENTAGON"
20 PRINT "WHAT ARE THE MEASURES OF FOUR ANGLES?"
30 INPUT MA, MB, MC, MD
40 ME = 540 - (MA+MB+MC+MD)
50 PRINT "THE FIFTH ANGLE MEASURES"; ME; "DEGREES."
60 END
```

Lesson: 36

AIM: How do we solve equations using inverse operations?

PERFORMANCE OBJECTIVES: The student will be able to...

1. solve equations of the form $x \pm a = b$ by addition or subtraction;
2. solve equations of the form $ax = b$ by division.
3. solve equations of the form $\frac{x}{a} = b$ by multiplication;

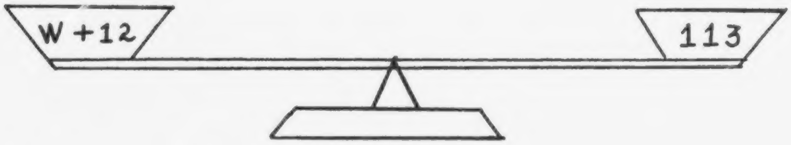
VOCABULARY: equation, solution, inverse operation, variable, coefficient

MOTIVATION:

After a diet, Mary weighs 113 pounds. This is 12 pounds more than Jennifer's weight. What is Jennifer's weight? (Note: In this lesson, the applications are contained within the development.)

DEVELOPMENT:

1. Ask, "How do we represent the words in this problem symbolically?"
   Let W = Jennifer's weight.
   $W + 12 = 113$
2. Develop a procedure for solving $W + 12 = 103$.
   a) Ask the students to think of an equation as a scale.



   b) Ask, "How can we get W alone?" (subtract 12)

c) Ask, "How do we keep the scale in balance?" (Subtract 12 from the other side.)

$$\begin{array}{rcl} W + 12 & = & 113 \\ \underline{\quad -12} & & \underline{-12} \\ W & = & 101 \end{array}$$

Jennifer weighs 101 pounds.

d) Ask, "How do we check the solution to the equation?" Ask the students to substitute into the original equation.

Check

$$\begin{array}{rcl} W + 12 & = & 113 \\ 101 + 12 & \overset{?}{=} & 113 \\ 113 & = & 113 \end{array}$$

Ask, "How can we check that 101 solves the verbal problem as well as our equation?" Ask the students to reread the problem and evaluate their solution.

3. Application 1: Solve algebraically and check.
    - a) $X + 157 = 523$
    - b) $14.6 + Y = 19.2$
    - c) $78.24 = X + 25.42$

4. Ask, "What is the procedure for solving $X - 15 = 80$? What is the variable? What has been subtracted from X? What is the opposite of subtracting 15? How do we keep the balance?"

   Elicit:

$$\begin{array}{rcl} X - 15 & = & 80 \\ \underline{\quad +15} & & \underline{+15} \\ X & = & 95 \end{array}$$

Check:

$$\begin{array}{rcl} X - 15 & = & 80 \\ 95 - 15 & \overset{?}{=} & 80 \\ 80 & = & 80 \end{array}$$

5. Application 2: Solve algebraically and check.
    - a) $W - 127 = 834$
    - b) $18.9 = Y - 6.5$
    - c) $X - 11.82 = 6.44$

6. Ask, "What is the procedure for solving $5X = 35$? What is the variable? By what number has X been multiplied?" Tell the students that the number that multiplies the

variable is called the coefficient. Ask, "What is the opposite of multiplying by 5? How do we keep the balance?"

Elicit:

$\frac{5X}{5} = \frac{35}{5}$ Check: $5X = 35$

$x = 7$ $5 \cdot 7 \overset{?}{=} 35$

$35 = 35$

7. Application 3: Solve algebraically and check.
   a) $3Y = 189$ c) $165 = 10Z$
   b) $6W = 72$
8. Ask, "What is the procedure for solving $\frac{X}{4} = 7$? What is the variable? By what number has X been divided? What is the opposite of dividing by 4? How do we keep the balance?"

Elicit

$\frac{X}{4} = 7$ Check: $\frac{X}{4} = 7$

$\frac{X}{4} . 4 = 7 . 4$ $\frac{28}{4} \overset{?}{=} 7$

$X = 28$ $7 = 7$

9. Application 4: Solve algebraically and check.
   a) $\frac{Y}{2} = 15$ c) $\frac{X}{2.5} = 3.6$
   b) $10 = \frac{W}{3}$
10. Ask, "How do we know which procedure to use?"
    Application 5: Solve algebraically and check.
    a) $\frac{Y}{8} = 3$ c) $2A = 9$
    b) $W - 8.2 = 7.8$ d) $412 = B + 72$

SUMMARY:

How do you know which inverse operation to use for a given equation?

HOMEWORK:

Solve algebraically and check.

1. $X - 8 = 9$
2. $N + .5 = 2.9$
3. $\frac{Y}{8} = 3$
6. $8B = 72$
7. $T - 1.05 = 2.3$
8. $3 = \frac{A}{5}$

4. $W + \frac{1}{2} = \frac{3}{4}$

5. $182 + Z = 237$

9. $10.8 = 1.2X$

10. $12.45 = N - 5.35$

HOMEWORK SOLUTIONS:

1. $X = 17$
2. $N = 2.4$
3. $Y = 24$
4. $W = \frac{1}{4}$
5. $Z = 55$
6. $B = 9$
7. $T = 3.35$
8. $15 = A$
9. $9 = X$
10. $17.8 = N$

Lesson: 37

AIM: How do we solve verbal questions using equations?

PERFORMANCE OBJECTIVES: The student will be able to...

1. represent unknown quantities in a verbal problem with a variable;
2. translate the words in a problem into an equation which can be used to solve the problem;
3. solve for and check the solution of a verbal problem.

VOCABULARY: legend, sum, increased by, more than, difference, decreased by, diminish, less than, product, quotient, multiplied by, divided by, times, twice, double, dictionary

MOTIVATION:

The price of hamburgers was increased by 12 cents. The new price is $1.05. What was the original price?

DEVELOPMENT:

Although many students will solve the motivation problem arithmetically, say to the students that computer solutions require representing quantities symbolically with variables.

1. Ask, "What quantity should be represented with a variable?"

   Let P = the original price of a hamburger.
   Say to the students that this statement is necessary to identify what quantity is represented by the variable. We call such a statement the legend or dictionary.

2. Ask, "How do we translate the words in the problem into an equation?" Elicit from the students words that can be translated into each mathematical symbol; e.g.,

   +: plus, add, sum, increased by, more than...
   -: minus, subtract, difference, decreased by, less than, diminish...
   x: times, multiplied by, product, of, twice, double...
   ÷: divided by, quotient...
   =: equal to, is...

Ask the students to translate the words in the motivation problem into an equation as follows.

$p + .12 = 1.05$

3. Review the procedure for solving equations.

$$\begin{array}{rcr} p + .12 & = & 1.05 \\ \underline{\quad -.12} & & \underline{-.12} \\ p & = & .93 \end{array}$$

Check $p + .12 = 1.05$

$$.93 + .12 \stackrel{?}{=} 1.05$$

$$1.05 = 1.05$$

4. Ask the students to write out the answer to the verbal problem in words; e.g.,
   The original price of a hamburger was 93¢.

5. Ask the students to verify (check) the answer by referring to the original words of the problem; e.g.,
   .93 increased by .12 should be 1.05.

APPLICATION:

Solve each by translating into an equation.

1. For a sale the regular price of a dress was decreased by \$3.49. The sale price is now \$17.99. What was the regular price?
2. The winner of a race had a time of 10.25 seconds, which is .38 seconds less than the 2nd place runner. What was the time of the 2nd place runner?
3. A bank charges 12¢ for each check written. If the monthly charge was \$1.80 how many checks were written?
4. Fifty-nine students signed up for intramural basketball. How many teams of 7 players can be formed?
5. 1K of computer memory is 1024 bytes. How many bytes of memory does a computer with 128K of memory have?

SUMMARY:

What questions should we ask ourselves when solving verbal problems algebraically?

HOMEWORK:

Solve using an equation. Verify your solution by referring back to the words of the problem.

1. School lunch costs 95¢ per day. How many lunches can you buy for $11.50?
2. Mary lost 12 lbs. on a diet. Her new weight is 109 lbs. What did she weight before the diet?
3. Jason scored 17 points, which is 5 more than Reggie scored. How many points did Reggie score?
4. There are 8 slices per pizza pie. How many pies should be bought for a party so there are at least 55 slices.
5. If Sam earns $3.35 per hour, how much will he earn in 22 hours?

HOMEWORK SOLUTIONS:

1. Twelve lunches can be bought for $11.50.
2. Mary's weight before the diet was 121 lbs.
3. Reggie scored 12 points.
4. Seven pizza pies should be bought.
5. Sam will earn $73.70 in 22 hours.

Lesson: 38L

AIM: How do we code a program to have the computer solve for X, all equations of the form X + B = C?

PERFORMANCE OBJECTIVES: The student will be able to...

1. determine and use the algorithm X = C - B to solve for X, equations in the form of X + B = C;
2. develop a plan which incorporates the algorithm X = C - B;
3. design a planned output;
4. write a program from the plan and planned output;
5. run the program to check the algebraic solutions of equations.

MOTIVATION:

Solve using an equation.

Jim is 3 inches taller than Bill. If Jim is 66" tall, how tall is Bill?

VOCABULARY: no new terms

DEVELOPMENT:

1. Review the procedure for solving verbal problems.

2a. Ask, "How can we use the computer to solve such problems?" Say to the students that since such problems can be solved using equations, we must develop an algorithm for solving the equations. Ask, "What equation was used to solve our motivation problem?" Ask the students to notice that the general form of the equation is X+B = C.

2b. Present the problem that follows for a computer solution.

> Problem: Enter values for B and C. Have the computer solve for X, all equations of the form X + B = C.

3. Ask the students to write the analysis that follows. Ask, "How do we solve equations of the form X + B = C for X?"

Analysis:

Use the parallel development that follows to derive the algorithm.

$$\begin{array}{lcr} X + 3 & = & 66 \\ \underline{\quad -3} & & \underline{-3} \\ X & = & 63 \end{array} \qquad\qquad \begin{array}{lcl} X + B & = & C \\ \underline{\quad -B} & & \underline{-B} \\ X & = & C - B \end{array}$$

4a. Ask, "How can we use this algorithm in a computer solution for X?" Elicit the following plan.

Plan

1) Enter values for B and C.
2) Calculate the solution: X=C-B.
3) Print X.
4) End.

4b. Ask the following questions to assist the students in writing the plan.

i) What information do we need to enter into the computer?
ii) What algorithm do we want the computer to use in order to find the solution?
iii) What value do we want the computer to display? In what memory cell will we find that value?

5. Ask the students to write a planned output reminding them of the importance of the title, prompt, and the explanation of the answer.

Planned Output:

```
TITLE       SOLVING EQUATIONS OF THE FORM X+B=C FOR X
PROMPT      ENTER B AND C.
INPUT       ?3, 66
ANSWER      X = 63
```

6. Ask the students to write the program.

Program

```
10 PRINT "SOLVING EQUATIONS OF THE FORM X+B=C FOR X"
20 PRINT "ENTER B AND C."
30 INPUT B,C
40 X = C-B
50 PRINT "X="; X
60 END
```

7. Ask the students to solve the following equations both algebraically and by tracing their programs.

a) $X + 782 = 1094$ c) $15.75 = X + 3.5$

b) $X + 6.2 = 9.1$

Ask, "What is the value of B? ...C?"

8. Ask the students to enter their programs and run them using the equations in the Applications so that they may verify that their program solves the problem.

APPLICATIONS:

1. Solve each equation algebraically and check it using your program.

   a) $X + 123 = 942$ c) $82.54 = X + 9.31$

   b) $X + 8.3 = 12.9$ d) $X + .24 = 1.2$

2. Write an algorithm and then incorporate it into a plan for a computer solution of equations of the form X-B=C for X.

HOMEWORK:

1. Solve algebraically and by tracing the program written in today's lesson.

   a) $X + 421 = 1246$ c) $X + 72.58 = 842.65$

   b) $8.45 = x + 3.2$ d) $X + 6\frac{1}{4} = 15\frac{1}{2}$

   When going over the homework assignment, ask, "How would we enter $6\frac{1}{4}$ and $15\frac{1}{2}$ into the computer?" Elicit that they would be entered in decimal form.

2. Design a planned output and then write the program to solve equations of the form X-B=C for X. Use the plan developed in today's lesson.

3. Solve each equation by tracing the program written in 2.

   a) $X - 315 = 243$ b) $18.2 = X- 9.3$

HOMEWORK SOLUTIONS:

1. a) $X = 825$ c) $X = 770.07$

   b) $X = 5.25$ d) $X = 9\frac{1}{4}$ or 9.25

2. Planned Output

```
SOLVING EQUATIONS OF THE FORM X-B=C FOR X
ENTER B AND C.
?3,8
X=11
```

Program

```
10 PRINT "SOLVING EQUAITONS OF THE FORM X-B = C FOR X"
20 PRINT "ENTER B AND C."
30 INPUT B,C
40 X = C + B
50 PRINT "X="; X
60 END
```

3. a) Screen

```
Run
SOLVING EQUATIONS
OF THE FORM
X-B = C FOR X
?315,243
X = 558
```

Memory

```
 B   C    X
315 243 243+315 =558
```

b) Screen

```
Run
SOLVING EQUATIONS
OF THE FORM
X-B = C FOR X
? 9.3,18.2
X = 27.5
```

Memory

```
  B    C     X
9.3  18.2 18.2+9.3 = 27.5
```

Lesson: 39L

AIM: How do we code a program to have the computer solve for X all equations of the form AX=B?

PERFORMANCE OBJECTIVES: The student will be able to...

1. determine and use the algorithm $X = \frac{B}{A}$ to solve for X, equations of the form AX=B;
2. develop a plan which incorporates the algorithm $X = \frac{B}{A}$;
3. design a planned output;
4. write a program from the plan and planned output;
5. run the program to check the algebraic solution of equations and verbal problems.

MOTIVATION:

Solve the problem using an equation.
Deborah earns $4 per hour. If her gross pay is $68, how many hours did she work?

VOCABULARY: gross pay

DEVELOPMENT:

1. Review with the students the procedure for solving the problem.

2a. Ask, "How can we use the computer to solve such problems?" Say to the students that since such problems can be solved using equations, we must develop an algorithm for solving the equations. Ask, "What equation was used to solve our motivation problem?" Ask the students to notice that the general form of the equation is AX=B.

2b. Present the problem that follows for a computer solution.

Problem: Enter values for A and B. Have the computer solve for X, all equations of the form AX=B.

3. Ask the students to write the analysis that follows. Ask, "How do we solve equations of the form AX=B for X?"

Analysis:

Use the parallel development that follows to derive the algorithm.

$$\frac{4X}{4} = \frac{68}{4} \qquad \frac{AX}{A} = \frac{B}{A}$$

$$X = 17 \qquad X = \frac{B}{A}$$

4a. Ask, "How do we use this algorithm in a computer solution for X?" Elicit the following plan.

Plan

1) Enter values for A and B.
2) Calculate the solution: $X = \frac{B}{A}$.
3) Print X.
4) End.

4b. Ask the following questions to assist the students in writing the plan.

i) What information do we need to enter into the computer?
ii) What algorithm do we want the computer to use in order to find the solution?
iii) What value do we want the computer to display? In what memory cell will we find that value?

Ask the students to create a planned output, reminding them of the importance of the title, prompt and an explanation of the answer.

Planned Output

TITLE	SOLVING EQUATIONS OF THE FORM AX=B FOR X
PROMPT	ENTER A AND B.
INPUT	?4,68
ANSWER	X=17

5. Ask the students to write the program.

Program

```
10 PRINT "SOLVING EQUATIONS OF THE FORM AX=B FOR X"
20 PRINT "ENTER A AND B."
40 X = B/A
50 PRINT "X=":X
60 END
```

6. Ask the students to solve the following equations both algebraically and by tracing their programs.
    - a) $7X = 63$
    - b) $4X = 26$
    - c) $12.6 = 3X$

   Ask, "What is the value of A? ...B?"

7. Ask the students to enter their programs and run them using the above equations.

APPLICATIONS:

1. Solve algebraically and check using your program.
    - a) $16X = 112$
    - b) $1.5X = 30$
    - c) $84 = 1.2X$
    - d) $.25X = 17.5$
2. Write an algorithm and incorporate it into a plan for a program to solve equations of the form $\frac{X}{A} = B$ for X.

SUMMARY:

Write the equations that follow on the chalkboard and then ask the questions below.

- a) $9X = 3186$
- b) $X-20 = 38.9$
- c) $x=41.7 = 86.4$

1. Which program written either yesterday or today would you use to solve each equation?
2. What data qould you need to enter into the computer for each equation?
3. What would be the output for each equation?
4. Trace the program you have selected for each. Does it solve the equation?

HOMEWORK:

1. Solve algebraically and by tracing the program written in today's lesson.
    - a) $25X = 950$
    - b) $4.2X = 32.76$
    - c) $5X = 32$
    - d) $12X = 1260$
2. Design a planned output and then write the program to solve all equations of the form $\frac{X}{A} = B$ for X. Use the

plan developed in today's lesson.

3. Solve each equation by tracing the program written in 2.

   a) $\frac{X}{3} = 10$ b) $\frac{X}{5} = 2.4$

HOMEWORK SOLUTIONS:

1. a) X = 38 c) X = 6.4

   b) X = 7.8 d) X = 105

2. Planned Output

```
SOLVING EQUATIONS OF THE FORM X/A=B FOR X
ENTER A AND B.
?7,6
X=42
```

Program

```
10 PRINT "SOLVING EQUATONS OF THE FORM A/X=B FOR X"
20 PRINT "ENTER A AND B."
30 INPUT A,B
40 X = B*A
50 PRINT "X=";X
60 END
```

3. a) Screen

```
RUN
SOLVING EQUATIONS OF THE FORM X/A=B FOR X/A = B FOR X
ENTER A AND B.
? 3,10
X = 30
```

   b) Screen

```
RUN
SOLVING EQUATIONS OF THE FORM X/A = B FOR X
ENTER A AND B.
? 5, 2.4
X = 12
```

Lesson: 40

AIM: How do we revise the plan for the solution of equations of the form AX = B so that we can avoid a division by zero message?

PERFORMANCE OBJECTIVES: The student will be able to...

1. use flowcharting symbols in writing a pictorial plan to solve a problem;
2. demonstrate an understanding of the meanings of relation symbols;
3. trace a flowchart.

VOCABULARY: flowchart, flowcharting symbols, relation symbols, solution, sum, product, difference, quotient

MOTIVATION:

A student is using the program below to solve for X, equations of the form AX = B.

Program

```
10 INPUT A,B
20 X = B/A
30 PRINT X
40 END
```

If the student enters the values A = 0 and B = 6, would the computer find a solution?

DEVELOPMENT:

1. If a computer is available, ask a student to enter the program and run it. (Otherwise, discuss what follows with the students.) Ask all the students to notice that a solution is not found but instead a division by zero message is displayed. Ask, "Why does this message appear?" Discuss with the students that when A = 0 and B = 6, in line 20: X = B/A, X = 6/0
and division by zero is impossible! You might wish to ask the students to "prove it." Say that suppose

division by zero is possible and the quotient for 6/0 is X. Ask, "How would we check the answer?" Elicit that X(0) = 6 has no solution; therefore, division by zero is not possible.

2. Ask, "How can we avoid the division by zero message?" Say that we must write a revised plan so that our program includes a check for A = 0.

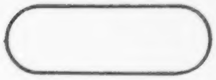
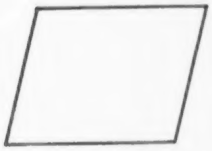
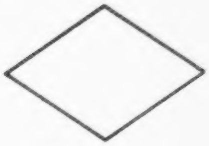
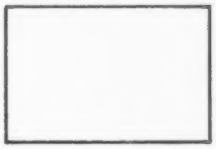
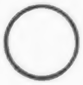
3. Elicit the revised plan from the students.

   Revised Plan (solving for X all equations of the form AX = B)

   1) Put in values for A and B.
   2) Check to see if A = 0.
      - a) If A=0, print 'A' CANNOT BE ZERO. Then go to 1.
      - b) If A ≠ 0, continue.
   3) Calculate: X = B/A.
   4) Print X.
   5) End.

4. Say that when plans involve checks like this one, they become more difficult to translate into programs. It is helpful to be able to write these plans in a pictorial form called a flowchart. Distribute the following table of flowcharting symbols.

Flowcharting Symbols

SYMBOL	MEANING	
	START/END	The computer is to start or end a program.
	INPUT/OUTPUT	The user is to put information into the computer or the computer is to display (print) information.
	DECISION	The computer is to decide whether a statement is true or false.
	ASSIGNMENT (CALCULATION)	The computer calculates and assigns to the variable on the left side of the equal sign the value from the right side.
	CONNECTOR	The computer scientist wants to come from or go to another part of the flowchart.

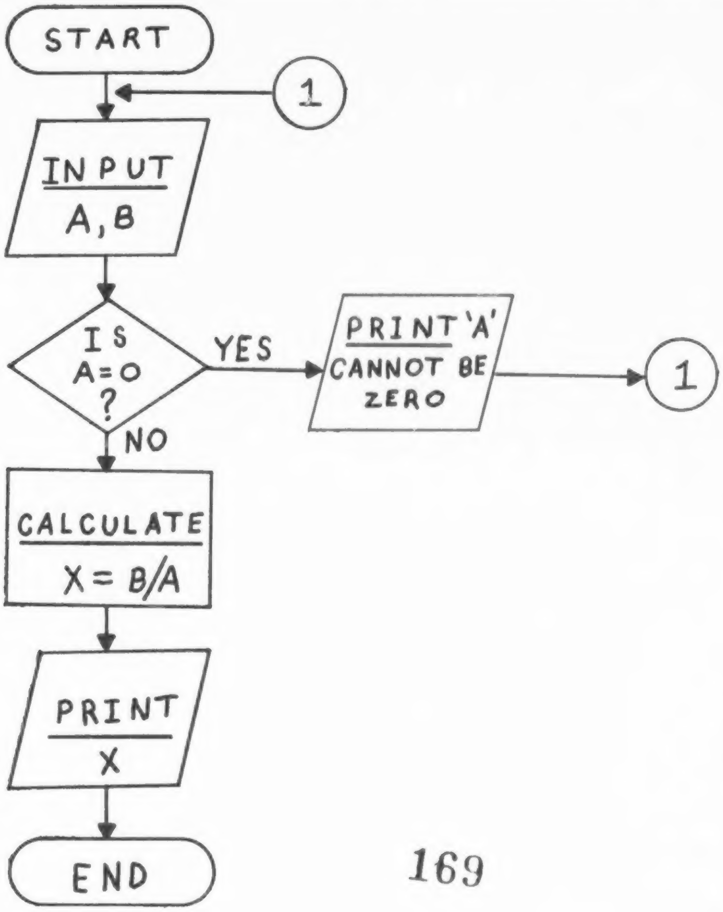
5. Say that when the computer is asked to make a decision, it must compare or find the relationship between two values. In mathematics and in BASIC, we use relation symbols for this purpose. Distribute or write on the chalkboard a table of relation symbols as follows.

Relation Symbols

Mathematics	Meaning	BASIC
$=$	is equal to	=
$>$	is greater than	>
$<$	is less than	<
$\neq$	is not equal to	<>
$\geq$	is greater than or equal to	>=
$\leq$	is less than or equal to	<=

6. Say, "Let us now use our symbols to flowchart our revised plan."

Flowchart (solving for X all equations of the form AX = B)


START
1
INPUT
A, B
IS
A=O
?
YES
PRINT 'A'
CANNOT BE
ZERO
1
NO
CALCULATE
X = B/A
PRINT
X
END

7. Ask the following questions as you develop the flowchart.
   a) Which flowcharting symbol do we use to indicate that we are beginning?
   b) Which flowcharting symbol do we use when we want to enter information?
   c) Which flowcharting symbol do we use when we want the computer to "make a decision"?

   Stress that from now on the YES answer to the decision will be written on the right and the NO answer will be written below the diamond.

   Working on the NO first, ask the following questions.
   d) Which flowcharting symbol do we use to indicate a calculation?
   e) Which flowcharting symbol do we use when we want the computer to display information?
   f) Which flowcharting symbol do we use to indicate that we are done?

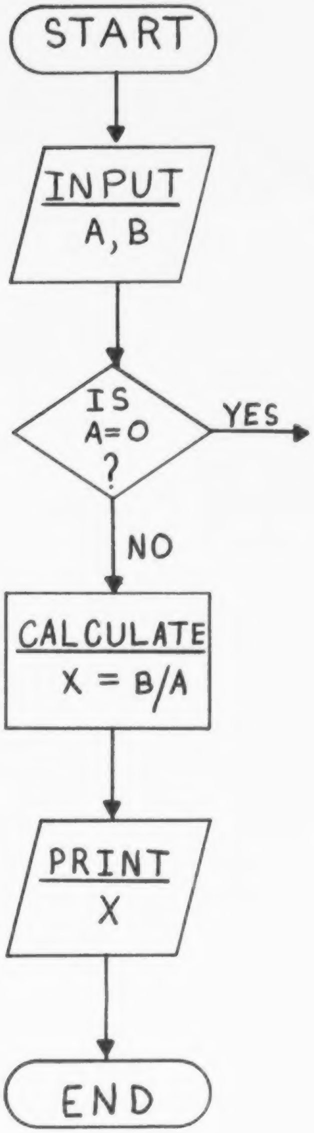
   Working on the YES, ask the following questions.
   g) Which flowcharting symbol do we use when we want the computer to display information in the form of a message?
   h) Which flowcharting symbol will enable us to join this part of the flowchart with the beginning so that we can give the user another chance to INPUT a nonzero value for A?

8. Say, "Before writing a program from a flowchart, we will trace the flowchart to see if it solves the problem. We will use the following steps."
   1) List in table form the names of all memory cells (variables) used in the flowchart.
   2) Follow the arrows in the flowchart.
   3) Perform all tasks.
   4) Record all the information in the table.

9. Ask the students to trace the flowchart to determine if it solves for X all equations of the form AX = B. Before beginning the trace, ask the students to determine the solutions of some equations (or use those from the

previous lesson); then ask them to see if the flowchart gives them the same solutions.

10. Elicit answers to the following questions.
What solution should the computer get for X when A = .2 and B = 6; i.e., when the equation is .2X = 6?



A	B	X

What are the names of all memory cells (variables) used in the flowchart?

A	B	X
.2	6	

What number will the computer store in memory cell A? ...memory cell B?

Check memory cell A and ask, "Is the number in A (.2) equal to zero?" (NO) Say that since the answer is NO, they are to follow the arrow down.

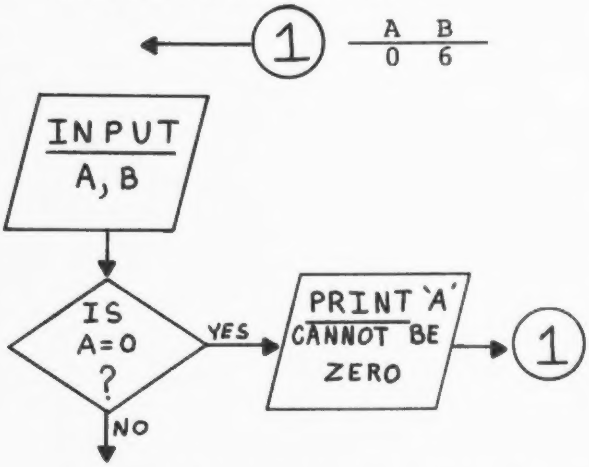
A	B	X
.2	6	6/.2
		30

What value does the computer calculate and store in memory cell X?

What number does the computer display? (30)

Since the solution should be 30, the flowchart solves the problem when the value entered for A is nonzero.

Now ask the students to see what happens if the user enters a zero for A.



Ask, "What number is stored in memory cell A? ...memory cell B?"

Check memory cell A. Ask, "Is the number in A equal to zero?" We will follow the YES arrow to the right and have the computer print 'A' CANNOT BE ZERO.

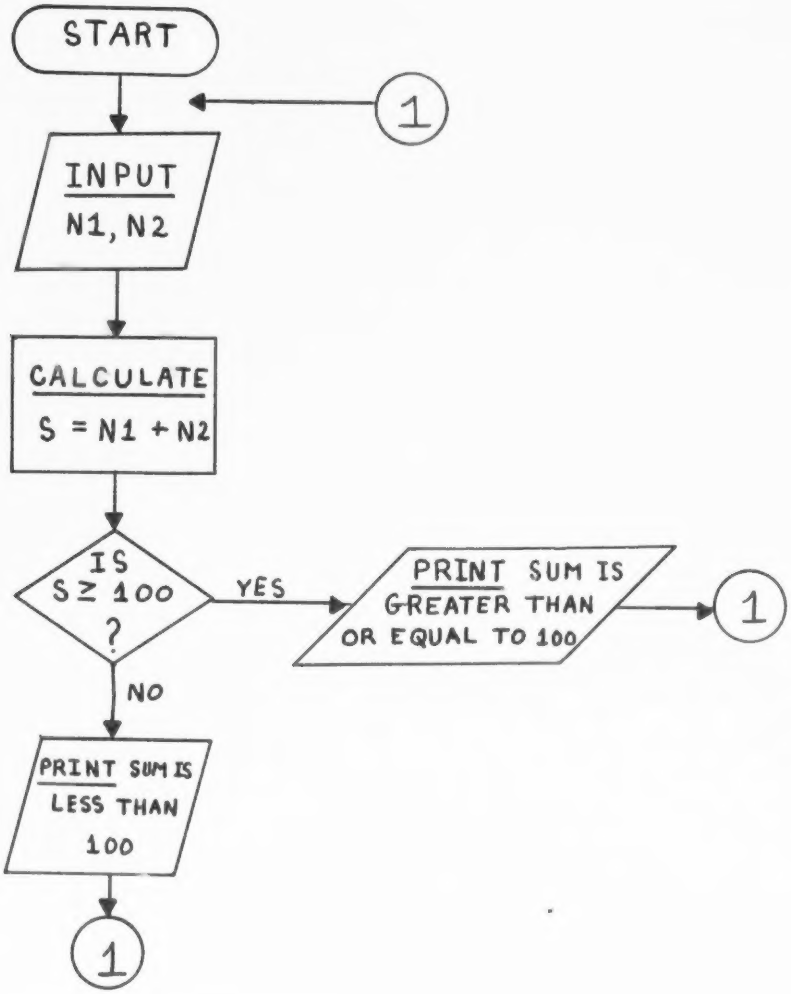
Follow the connector around, up and to the left. Ask, "Does the flowchart allow us to calculate when A = 0?" Elicit that it does not and that is exactly what we want. Conclude that since we have checked every part of the flowchart, our trace is complete.

APPLICATIONS:

1. Write each using flowcharting symbols.
   a) PRINT P
   b) IS S<=0?
   c) INPUT S
   d) R = A*B
2. Determine whether each of the following statements is true or false. (The symbols are written in BASIC.)
   a) (8+10)/2 > 8 + 10/2
   b) 2 + 3*4 = 2*3 + 4
   c) 12 + 4 <= 4*4
   d) 3↑4 <> 4↑3
3. Flowcharts for three problems follow. Determine if each flowchart solves the problem. Use the test data along with the procedure outlined for tracing a flowchart. If the flowchart does not solve the problem, correct it so that it will.
   a) Problem: Enter two numbers. Determine if their sum is greater than or equal to 100.

   Test Data: (i) 82,29
   (ii) 49,50

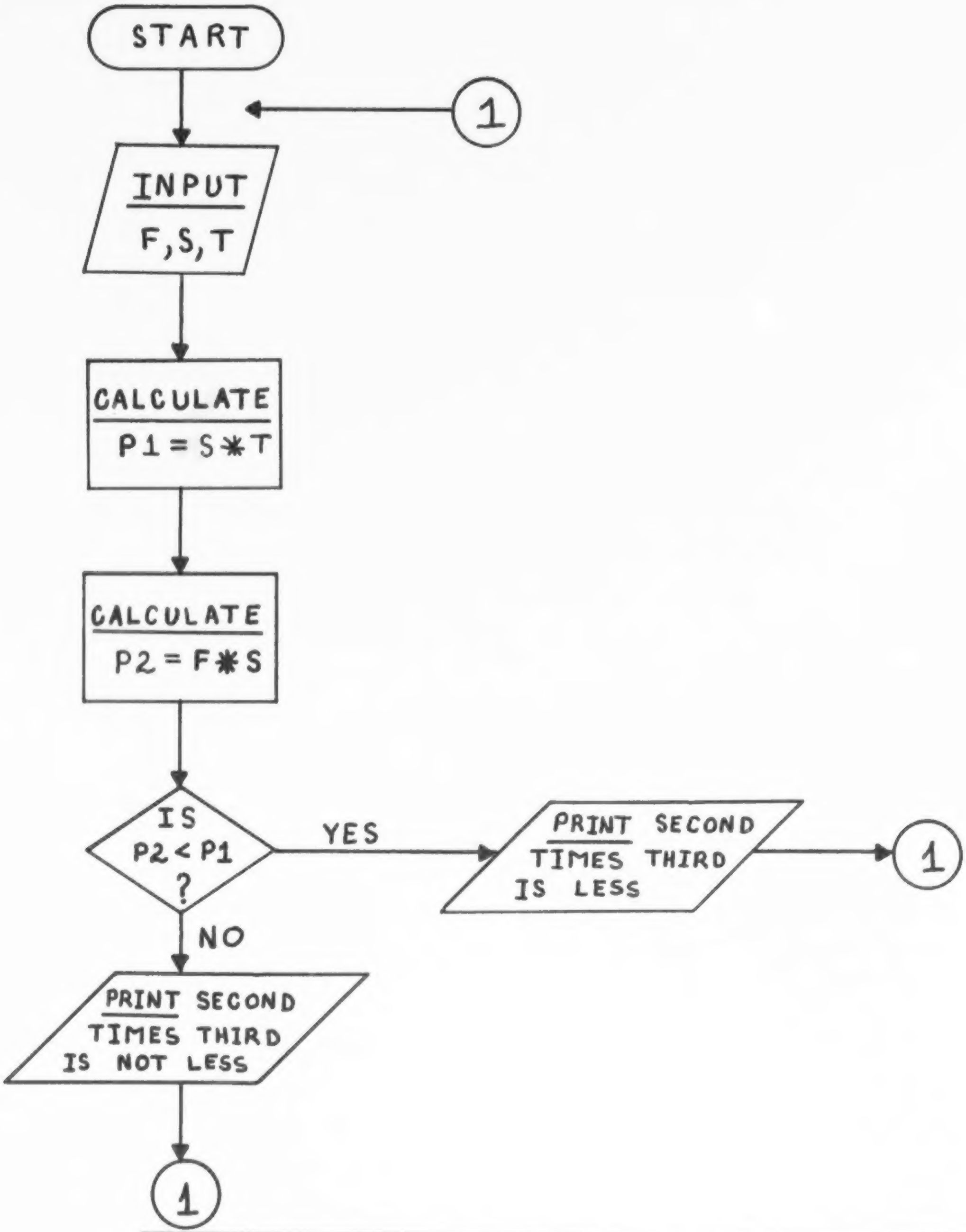
Flowchart


START
INPUT
N1, N2
CALCULATE
S = N1 + N2
IS S ≥ 100 ?
YES
PRINT SUM IS GREATER THAN OR EQUAL TO 100
1
NO
PRINT SUM IS LESS THAN 100
1

b) Problem: Enter three numbers. Determine if the product of the second and third numbers is less than the product of the first and second.

Test Data: (i) 4,6,5
(ii) 10,7,4

Flowchart


START
1
INPUT
F,S,T
CALCULATE
P1 = S*T
CALCULATE
P2 = F*S
IS
P2 < P1
?
YES
PRINT SECOND
TIMES THIRD
IS LESS
1
NO
PRINT SECOND
TIMES THIRD
IS NOT LESS
1

c) Problem: Enter two different numbers. Have the computer display them in descending order.

Test Data: (i) .3,.29

(ii) .09,.1

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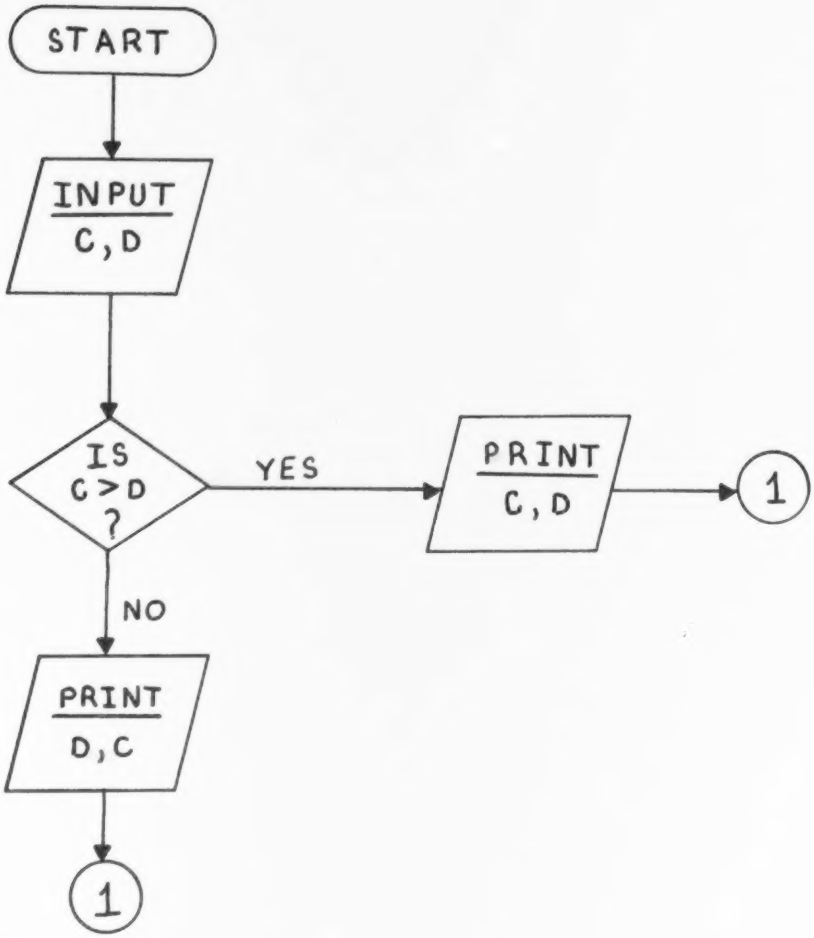
Flowchart


START
INPUT
C, D
IS
C > D
?
YES
PRINT
C, D
1
NO
PRINT
D, C
1

SUMMARY:

1. What is a flowchart?
2. How do we trace a flowchart?

HOMEWORK:

1. Solve each equation for x. Check your result.
    a) 3x = 1.5 c) x + 87 = 435.2
    b) 1.5x = 6 d) x - 3.5 = 84
2. Determine whether each of the following statements is true or false. (The symbols are written in BASIC.)
    a) `4 3 > 4*3` b) `8*3 + 8*4 <> 8*(3+4)`

3. Write each of the following using flowcharting symbols.

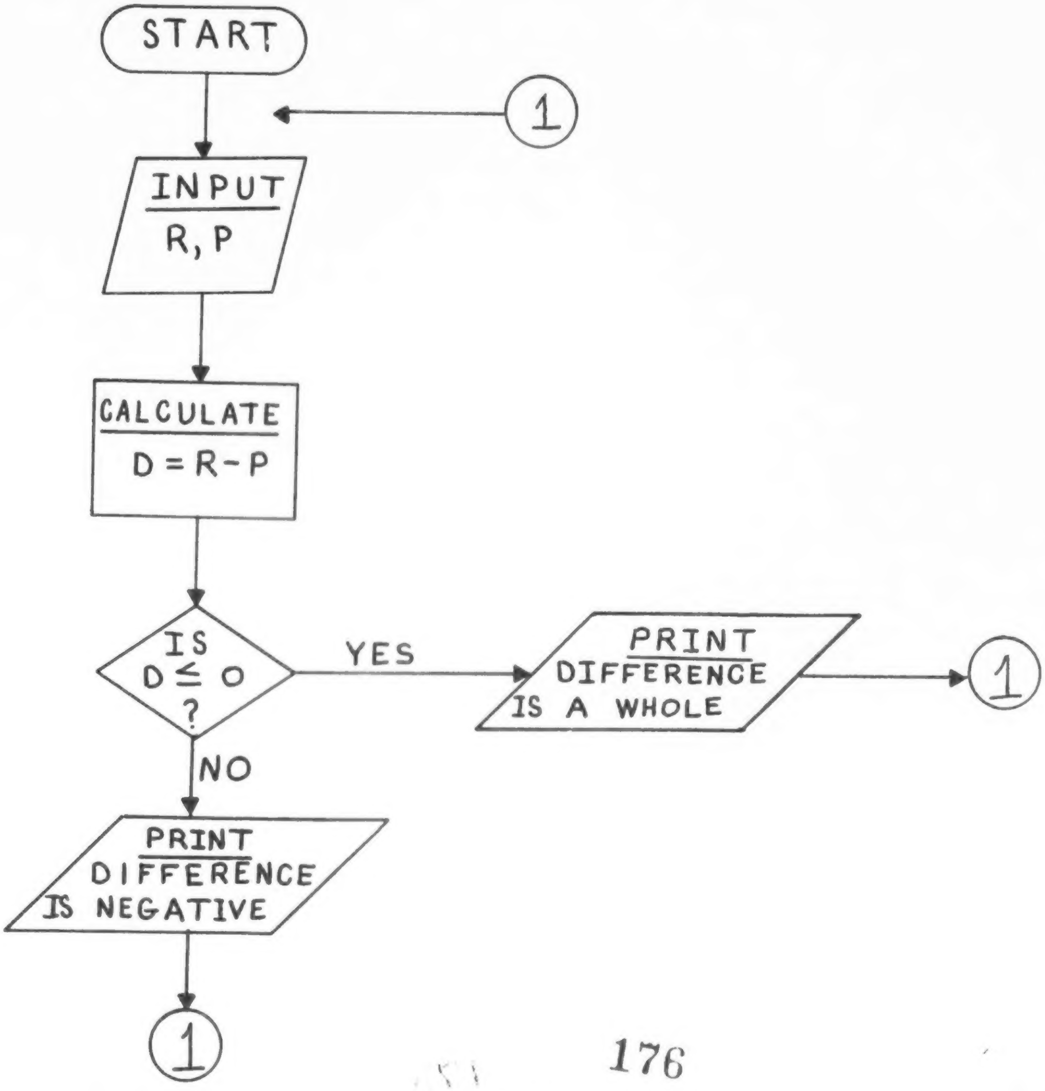
   a) INPUT X,Y
   b) T = 3*Y
   c) IS X < Y?
   d) PRINT "NONZERO PLEASE!"

In problem 4 which follows, determine if the flowchart solves the problem. Use the test data along with the procedure outlined for tracing a flowchart. If the flowchart does not solve the problem, correct it so that it will.

4. Problem: Enter two numbers. Determine if their difference is greater than or equal to zero.

   Test Data: (i) 8,3
   (ii) 3,8

Flowchart

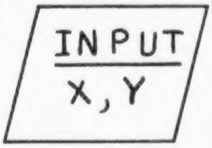

START
1
INPUT
R, P
CALCULATE
D = R - P
IS
D ≤ 0
?
YES
PRINT
DIFFERENCE
IS A WHOLE
1
NO
PRINT
DIFFERENCE
IS NEGATIVE
1

HOMEWORK SOLUTIONS:

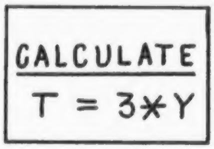
1. a) $x = 0.5$
   b) $x = 4$
   c) $x = 348.2$
   d) $x = 87.5$

2. a) true
   b) false

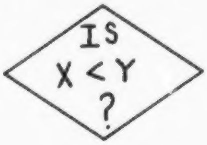
3. a)



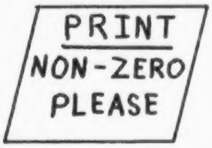
b)


CALCULATE
T = 3*Y

c)

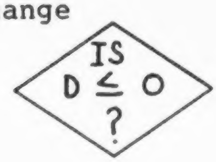


d)

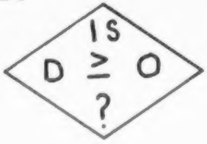

PRINT
NON-ZERO
PLEASE

4. The flowchart does not solve the problem.

Change



to



Lesson: 41

AIM: How do we program the computer to make decisions?
(See note on the bottom of page 170)

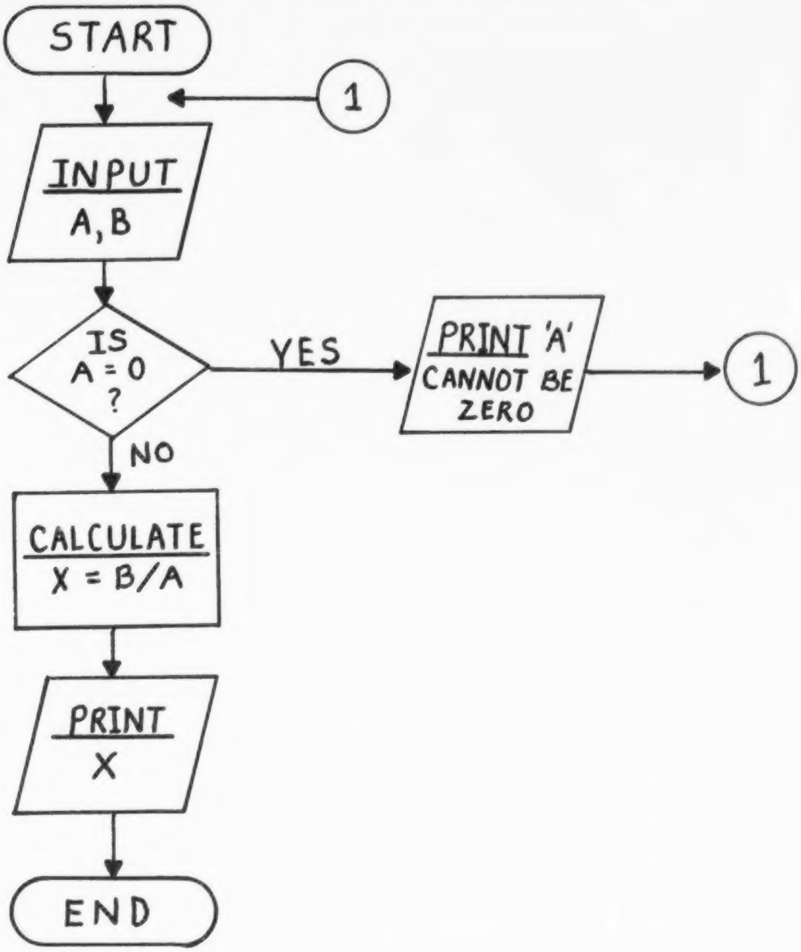
PERFORMANCE OBJECTIVES: The student will be able to...

demonstrate his/her understanding of the IF-THEN and GOTO statements by correctly showing outputs for programs.

VOCABULARY: IF-THEN instruction, GOTO instruction, looping, counting number

MOTIVATION:

Review the flowchart that we wrote yesterday.


START
1
INPUT
A,B
IS
A = 0
?
YES
PRINT 'A'
CANNOT BE
ZERO
1
NO
CALCULATE
X = B/A
PRINT
X
END

If we want to write a program using the flowchart as our plan, which two new computer ideas must we learn to program?

DEVELOPMENT:

1. Elicit answers to the question in the motivation...

   Idea #1: How do we get the computer to follow the connector (here ①)?

   Idea #2: How do we get the computer to make a decision (here to decide if A = 0)?

2. Say that to get the computer to follow the connector (Idea #1), we introduce the GOTO instruction.
   GOTO line number sends the computer to the line number; e.g., 50 GOTO 20 will send the computer back to line 20.

3. Say that to get the computer to make a decision (Idea #2), we introduce the IF-THEN instruction; e.g.,
   60 IF A = 0 THEN GOTO 90
   Say that if A = 0, the computer goes to line 90 in the program. Otherwise, the computer moves on to the next line in the program.

4. Stress the following for an **IF condition THEN instruction:**

   When the condition is true, the computer follows the instruction after the THEN.
   When the condition is false, the computer follows the instruction on the next line.

5a. Ask the students to consider the following program. Ask them to show the complete output when (i) N=7, (ii) N=-5.

Program

```
10 INPUT N
20 IF N<0 THEN GOTO 50
30 PRINT "THE NUMBER IS NOT NEGATIVE."
40 GOTO 60
50 PRINT "THE NUMBER IS NEGATIVE."
60 END
```

5b. Ask the students to trace the program as follows.

```
Screen                                    Memory
RUN                                         N
?7 ------------------------------------->   7
THE NUMBER IS NOT NEGATIVE.
RUN
?-5 ------------------------------------>  -5
THE NUMBER IS NEGATIVE.
```

i) When the number in memory cell N is 7, which is not less than zero, and the computer executes the instruction in line 20, it moves down to the next line, which is 30, and the computer displays the message there.

ii) When the number in memory cell N is -5, which is less than zero, and the computer executes the instruction in line 20, it moves across to the instruction after THEN, which sends the computer to line 50 and the computer displays the message there.

5c. Ask the students to complete Applications 1 and 2. If computers are available, ask the students to verify their results by running the programs. (The student must show the teacher the trace before he/she goes to the computer.)

6a. Present the program below. Ask the students to show the output when R = 10.

```
10 INPUT R
20 P = 2
30 PRINT P
40 P = P+2
50 IF P>R THEN GOTO 70
60 GOTO 30
70 END
```

6b. Ask the students to trace the program as follows.

Screen	Memory	
	P	R
RUN		
?10	~~2~~	10
2	2+2 = ~~4~~	
4	4+2 = ~~6~~	
6	6+2 = ~~8~~	
8	8+2 = ~~10~~	
10	10+2 = 12 > 10	

i) For lines 10 and 20, ask the students the following questions. "What appears on the screen as a result of the INPUT statement? What number is placed in memory cell R? ...in memory cell P?"

ii) Ask, "What appears on the screen when the computer executes line 30?"

iii) Say to the students that the statement P = P+2 in line 40 is read P becomes P+2. When this instruction is executed, the value in memory cell P becomes two larger and is stored back in memory cell P. Here, since the value stored is 2, P = 2+2 = 4 replaces 2 in memory cell P.

iv) Ask, "Which numbers does the computer compare in line 50?" Say that since 4 is not greater than 10, the computer goes down to line 60 (the next line). GOTO 30 sends the computer back to line 30. Ask, "What does the computer display then?"

v) Repeat lines 30 through 60 of the program until P becomes 12. Say that at that point, when the computer executes line 50, 12 is greater than 10. The computer follows the instruction after the THEN and goes to line 70 where the program ends.

vi) Say that the above procedure is sometimes called looping. Ask, "Why is that a good name?"

vii) Say that this program displays the first five even counting numbers. Ask, "How can we get instead all the counting numbers from 2 through 10 displayed?" (Elicit: Change P = P+2 to P = P+1.) Ask, "How can we get all the counting numbers from 1 through 100 displayed?" (Elicit: Change line 20 to P=1 and when the ? appears enter 100 for R instead of 10.)

viii) Ask the students to edit line 30 by adding a semicolon so that it reads 30 PRINT P; Ask the students to enter the change on the computer, if available. Ask, "What effect does the change have on the output?"

6c. Ask the students to complete Application 3. If computers are available, ask the students to verify their results by running the program.

APPLICATIONS:

For each of the following programs, show the output when A=10, B=15 and C=5.

1.

```
10 INPUT A
20 IF A<=0 THEN GOTO 50
30 PRINT "# IS POSITIVE."
40 GOTO 60
50 PRINT "# IS NOT POSITIVE."
60 END
```

2.

```
10 INPUT A,B
20 IF A <> B THEN 50
30 PRINT "SAME NOS."
40 GOTO 60
50 PRINT "DIFFERENT NOS."
60 END
```

3.

```
10 INPUT C
20 K=1
30 PRINT C
40 K = K+1
50 IF K>7 THEN GOTO 70
60 GOTO 30
70 END
```

Point out that GOTO is not needed after THEN.

SUMMARY:

What computer instruction is used when the computer is to make a decision?

HOMEWORK:

Show the complete output for each program, when G=5, H=15 and E=3.

1.

```
10 INPUT G,E
20 P = G*E
30 IF P>20 THEN GOTO 60
40 PRINT "VALUE IS";P
50 GOTO 70
60 PRINT P;"IS GREATER THAN 20"
70 END
```

2.

```
10 INPUT G,H,E
20 IF G = H/E THEN 50
30 PRINT "1ST IS NOT 2ND DIVIDED BY 3RD."
40 GOTO 60
50 PRINT "1ST IS 2ND DIVIDED BY 3RD."
60 END
```

HOMEWORK SOLUTIONS:

1. Screen

```
RUN
?5,3
VALUE IS 15
```

Memory

G	E	P
5	3	5*3=15

2. Screen

```
RUN
?5,15,3
1ST IS 2ND DIVIDED BY 3RD.
```

Memory

G	H	E
5	15	3

NOTE:
(The material in this lesson can be developed in one full period. To actually have the students run the program (s) will require a session in the computer laboratory.)

Lesson: 42

AIM: How do we write a program for the solution of equations of the form AX = B so that a division by zero message is avoided?

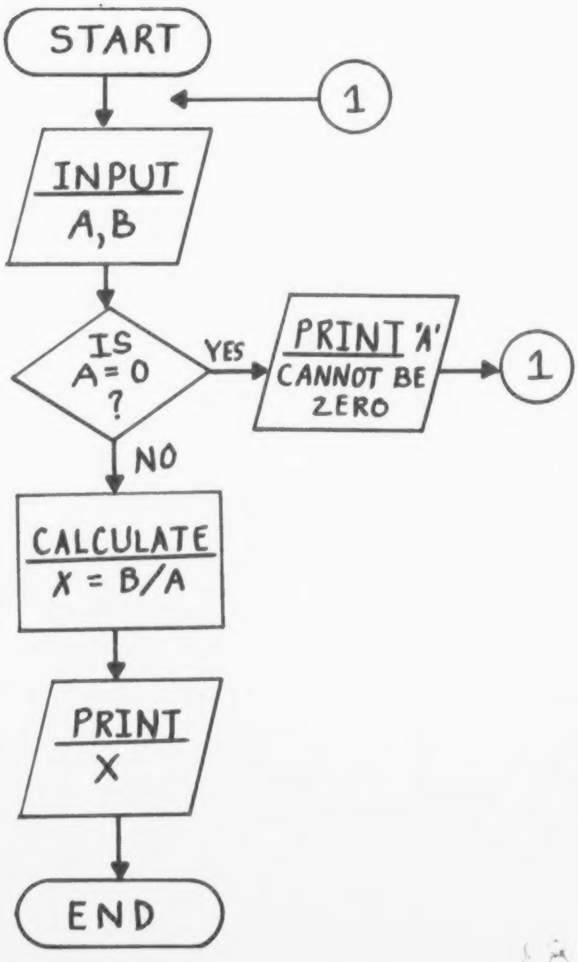
PERFORMANCE OBJECTIVES: The student will be able to...

demonstrate his/her understanding of how to use both a flowchart and a planned output in coding a program.

VOCABULARY: planned output, flowchart, solution, GOTO instruction, IF-THEN instruction, edit

MOTIVATION:

Write a program that will have the computer solve for X, all equations of the form AX = B. Use the flowchart and planned output below.



Planned Output

```
RUN
SOLVING EQUATIONS OF THE FORM AX=B
ENTER VALUES FOR A AND B.
?.2,6
THE SOLUTION IS
X=30
RUN
SOLVING EQUATIONS OF THE FORM AX=B
?0,6
'A' CANNOT BE ZERO.
ENTER VALUES FOR A AND B.
?
```

DEVELOPMENT:

Guide the students in writing the program as follows.

Program

```
10  PRINT "SOLVING EQUATIONS OF THE FORM AX=B"
20  PRINT "ENTER VALUES FOR A AND B."
30  INPUT A,B
40  IF A=0 THEN GOTO 90
50  X=B/A
60  PRINT "THE SOLUTION IS"
70  PRINT "X=";X
80  GOTO 110
90  PRINT "'A' CANNOT BE ZERO."
100 GOTO 20
110 END
```

1. Ask the students to program all the messages in the planned output that come before the question mark first. Point out the INPUT statement in the flowchart and the corresponding ? in the planned output. Ask the students to write lines 10 and 20 of the program. Ask, "How do we get a message displayed?" (PRINT with quotes.)

2. Say that they are to program straight down the flowchart. Ask them to program line 30. Ask them to check off the parts of the flowchart as they program them.

3. Ask, "What instruction do we use to have the computer make a decision?" For line 40, say that at this point we do not know where to send the computer because, as we go down the flowchart, we will be programming the NO part first, then the YES. We leave a blank space which will be filled in later. (At this point, line 40 of the program looks like this:

   40 IF A=0 THEN GOTO ______.)

   When typing in the program, the students should be reminded that they do not type the underline.

4. Ask the students to continue programming down the flowchart and write line 50.

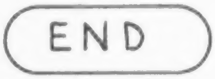
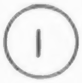
5. Say that when they get to a print statement in the flowchart, they should check the planned output to obtain the appropriate message to use in the program. Ask, "How do we get the message THE SOLUTION IS displayed? How do we get the message X= displayed before the computer displays a value for X?"

6. For line 80, point out that again a blank is left because we do not know at which line number the program will end.

7. Say, "Now that we have programmed down the flowchart, we are ready to go back and program the YES. We go back and fill in the number 90 (the next multiple of 10) in line 40 and program across the flowchart." Remind the students to check the planned output for the appropriate message when programming the PRINT statement.

8. Stress that every part of the flowchart, including the connector, must be programmed. Point out that the connector sends us back to INPUT A,B. This would be programmed GOTO 30. However, checking the planned output, we notice the message ENTER VALUES FOR A AND B. is displayed. We therefore write GOTO 20.

9. In our program, after the solution is displayed, the program ends as indicated in our flowchart. Ask, "What change do we have to make in our flowchart in order to have the computer go back to the beginning and wait for the new data to be entered?"

(Change  to  .)

Ask, "What change does this make in our program?" (We must edit line 80 to read: 80 GOTO 20.)

10. If computers are available, ask the students to run the original program and the program with the change in line 80. Ask the students to use the test data in the planned output and their own. Explain to the students how they can stop the execution of the revised program. (Note to the teacher: Check your manual to see how this is done since it is not the same procedure on all computers.)

11. Ask the students to complete the Application problem.

APPLICATION:

Revise the program written in today's lesson so that it will produce the output below.

```
RUN
SOLVING AX=B FOR X
TYPE IN VALUES FOR A AND B.
?.2,6
X=30
TYPE IN VALUES FOR A AND B.
?0,6
NONZERO VALUE FOR 'A' PLEASE.
TYPE IN VALUES FOR A AND B.
?
```

SUMMARY:

What things do we have to remember when writing programs from planned outputs and flowcharts?

Elicit:

1) Program all messages from the planned output that come before the "?" at the beginning of a line.
2) Program straight down the flowchart.
3) Program the NO before the YES.
4) When we come to a print statement in the flowchart, we must look at the planned output to see the appropriate message that will go with the PRINT statement in the program.

HOMEWORK:

1. Solve each equation. Check your results.
   a) $3x = 3192$ c) $6x = 0$
   b) $.3x = 1.5$

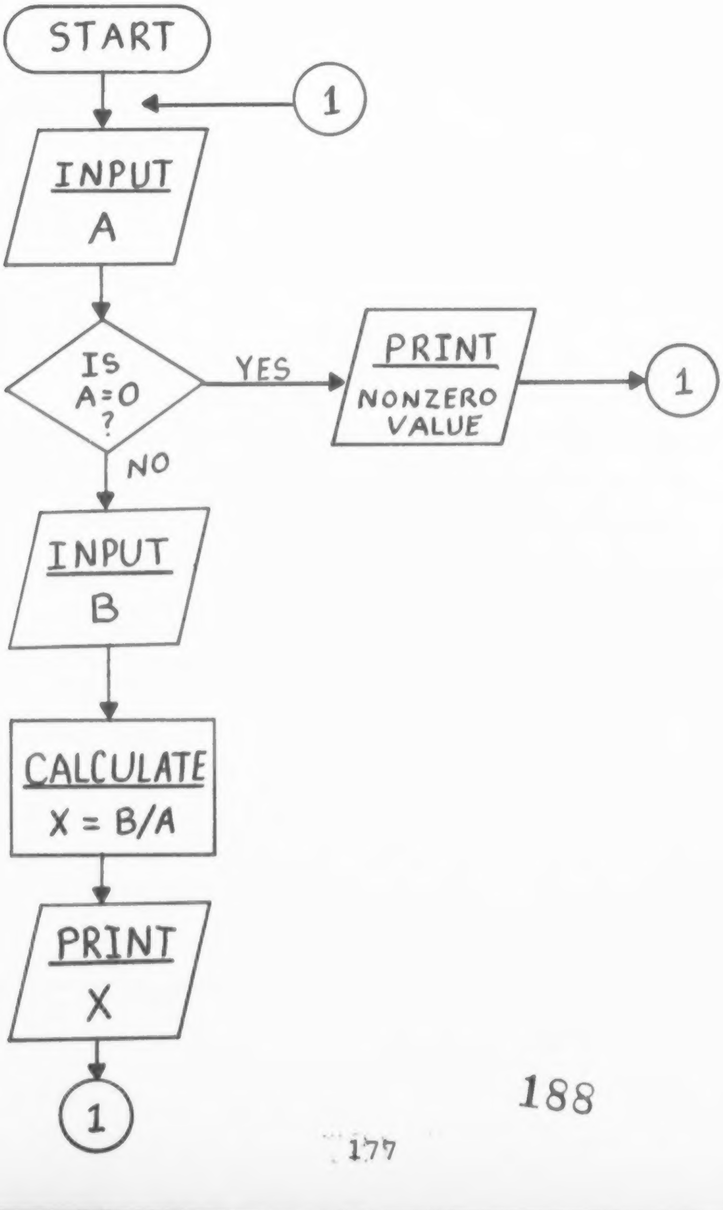
2. a) Revise the flowchart used in today's lesson so that the values for A and B are entered separately.
   b) Then revise the program so that it will produce the following planned output.

```
RUN
FINDING X FOR AX=B
ENTER A.
?.2
ENTER B.
?6
X IS 30 FOR THE EQUATION .2X=6
ENTER A.
?0
THE VALUE CANNOT BE ZERO.
?
```

HOMEWORK SOLUTIONS:

1. a) $x = 1064$ c) $x = 0$
   b) $x = 5$

2a)


START
1
INPUT
A
IS
A=0
?
YES
PRINT
NONZERO
VALUE
1
NO
INPUT
B
CALCULATE
X = B/A
PRINT
X
1

2b)

```
10  PRINT "FINDING X FOR AX=B"
20  PRINT "ENTER A."
30  INPUT A
40  IF A=0 THEN GOTO _100_
50  PRINT ENTER B."
60  INPUT B
70  X = B/A
80  PRINT "X IS";X;"FOR THE EQUATION";A;"X=";B
90  GOTO 20
100 PRINT "THE VALUE CANNOT BE ZERO."
110 GOTO 30
120 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

Lesson: 43L

AIM: How do we code a program from a flowchart and a planned output?

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate his/her understanding of how to use both a flowchart and a planned output in coding a program;
2. enter his/her coded solution to the problem;
3. run his/her coded solution to the problem, using appropriate test data;
4. correct any error in the program.

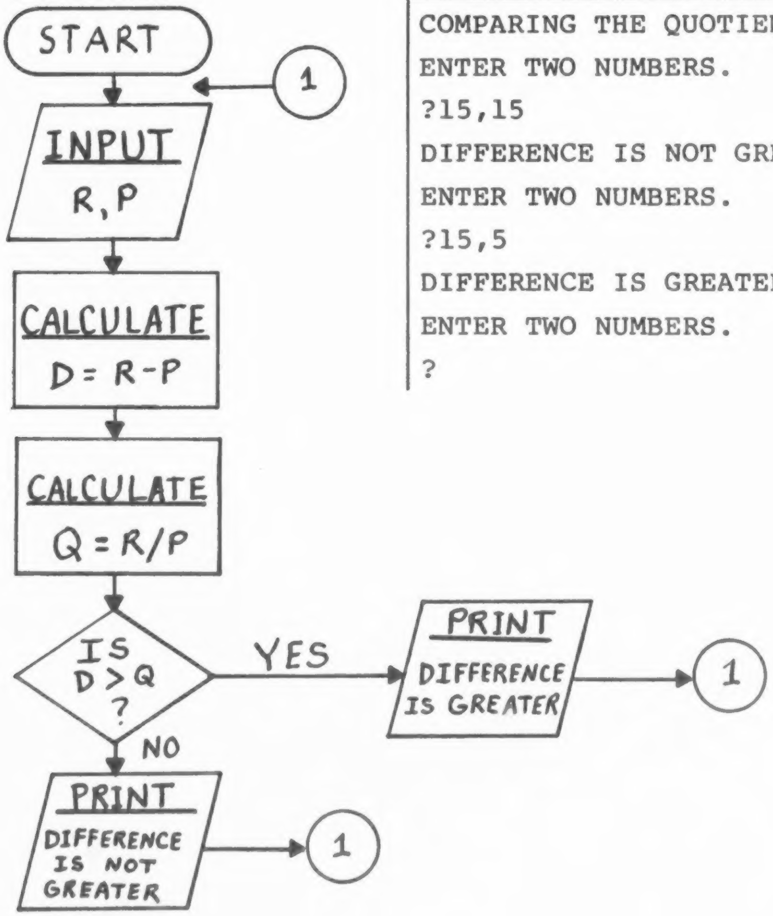
VOCABULARY: quotient, difference, product, sum, increase, double, triple

MOTIVATION:

For the problem, flowchart and planned output which follow, write a program.

Problem: Enter two numbers. Determine if the difference of the numbers is greater than the quotient.

Flowchart


START
1
INPUT
R, P
CALCULATE
D = R-P
CALCULATE
Q = R/P
IS
D > Q
?
YES
PRINT
DIFFERENCE
IS GREATER
1
NO
PRINT
DIFFERENCE
IS NOT
GREATER
1

Planned Output

```
COMPARING THE QUOTIENT AND THE DIFFERENCE
ENTER TWO NUMBERS.
?15,15
DIFFERENCE IS NOT GREATER THAN THE QUOTIENT
ENTER TWO NUMBERS.
?15,5
DIFFERENCE IS GREATER THAN THE QUOTIENT
ENTER TWO NUMBERS.
?
```

DEVELOPMENT:

1. Ask the students to read the problem carefully. Review the definitions of "quotient" and "difference."
2. Ask the students to trace the flowchart with the data in the planned output. To obtain additional data, ask the students to:
   a) name two numbers whose difference is greater than their quotient.
   b) name two numbers whose difference is less than their quotient.
   c) name two numbers whose difference is equal to their quotient.

   When tracing the flowchart, ask the students, "Why don't we say, under the NO, DIFFERENCE IS LESS THAN THE QUOTIENT?"

3. Discuss the planned output. Point out the title, the instructions to the user and the explanation of the results. Ask the students to point out the part of the output that refers to the NO from the flowchart and the part that refers to the YES.

4. Remind the students of the following as they write the program.
   a) Program all messages before the ? first.
   b) Work down the flowchart.
   c) Program the NO before the YES.
   d) Check the planned output whenever they reach a print statement in the flowchart.

5a. Ask the students to code the program on paper and then type it into the computer. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

5b. Circulate around the laboratory giving assistance where needed.

5c. Group the students who are unable to write the program. Work through some of the coding with this group.

5d. Elicit the program below.

Program

```
10  PRINT "COMPARING THE QUOTIENT AND THE DIFFERENCE"
20  PRINT "ENTER TWO NUMBERS."
30  INPUT R,P
40  D=R-P
50  Q=R/P
60  IF D>Q THEN GOTO  90
70  PRINT "DIFFERENCE IS NOT GREATER THAN THE QUOTIENT"
80  GOTO 20
90  PRINT "DIFFERENCE IS GREATER THAN THE QUOTIENT"
100 GOTO 20
110 END
```

6. Ask the students who complete the problem presented in the motivation to do the Application problems.

APPLICATIONS:

1. Revise the program written in today's lesson so that it will produce the planned output below.

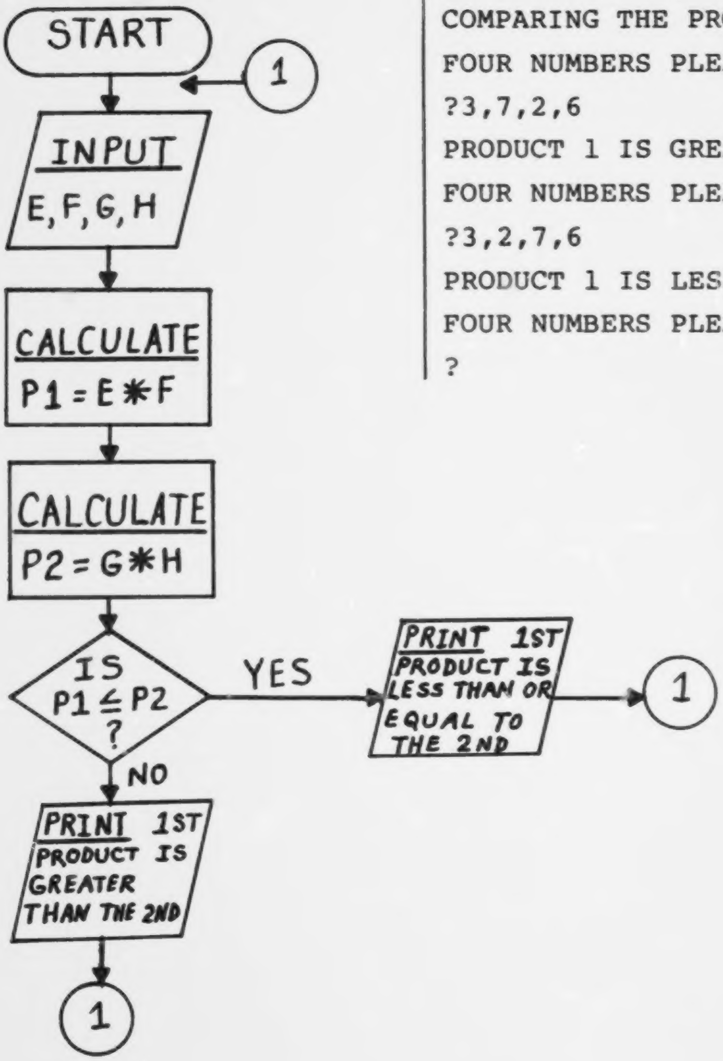
```
IS THE DIFFERENCE GREATER THAN THE QUOTIENT?
TYPE IN 2 NUMBERS.
?15,15
THE DIFFERENCE IS 0
THE QUOTIENT IS 1
THE DIFFERENCE IS NOT GREATER.
TYPE IN 2 NUMBERS.
?15,5
THE DIFFERENCE IS 10
THE QUOTIENT IS 3
THE DIFFERENCE IS GREATER.
TYPE IN 2 NUMBERS.
?
```

   As you circulate, assist the students by pointing out that the difference and the quotient are displayed whether the decision is YES or NO. Ask, "Is it better to program these messages before or after the decision instruction?"

2. For the problem, flowchart and planned output which follow:
   a) trace the flowchart;
   b) write a program that will produce the planned output;
   c) RUN the program using the test data from the planned output.

<u>Problem</u>: Enter four numbers. Determine if the product of the first and second numbers is less than or equal to the product of the third and fourth numbers.

Flowchart


START
1
INPUT
E,F,G,H
CALCULATE
P1 = E*F
CALCULATE
P2 = G*H
IS
P1 ≤ P2
?
YES
PRINT 1ST PRODUCT IS LESS THAN OR EQUAL TO THE 2ND
1
NO
PRINT 1ST PRODUCT IS GREATER THAN THE 2ND
1

Planned Output

```
COMPARING THE PRODUCTS
FOUR NUMBERS PLEASE.
?3,7,2,6
PRODUCT 1 IS GREATER THAN PRODUCT 2.
FOUR NUMBERS PLEASE.
?3,2,7,6
PRODUCT 1 IS LESS THAN OR EQUAL TO PRODUCT 2.
FOUR NUMBERS PLEASE.
?
```

SUMMARY:

What things do we need to keep in mind when coding a program from a flowchart and a planned output?

HOMEWORK:

1. Find the quotient when the difference between 4837 and 1342 is divided by 15.
2. Find the difference between the sum of 685 and 498 and the sum of 304 and 776.
3. From the sum of 803 and 797, subtract the product of 17 and 29.

4. Find the quotient when the sum of 407 and 385 is divided by their difference.

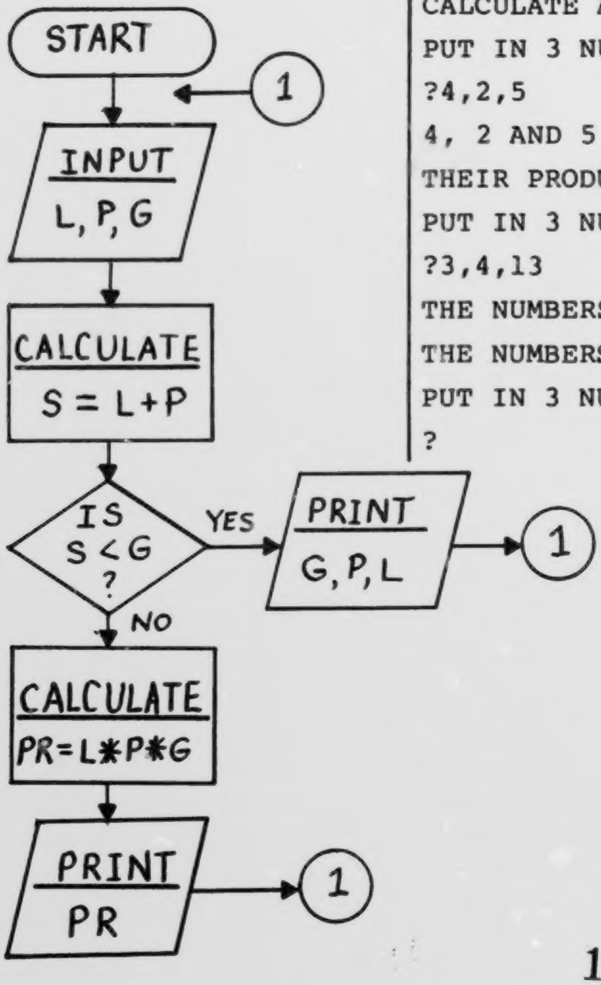
5. Find the product of 37 doubled and 5 tripled.

6. Which is greater and by how much, the product of 79 and 87 or the quotient when 96236 is divided by 14?

7. For the problem, flowchart and planned output which follow,
   a) trace the flowchart and
   b) write a program that will produce the planned output.

> Problem: Enter three numbers. If the sum of the first two numbers is less than the third number, display the numbers in reverse order. Otherwise, display the product of the numbers.

Flowchart



Planned Output

```
CALCULATE AND COMPARE
PUT IN 3 NUMBERS PLEASE.
?4,2,5
4, 2 AND 5 ARE THE NUMBERS.
THEIR PRODUCT IS 40
PUT IN 3 NUMBERS PLEASE.
?3,4,13
THE NUMBERS ARE 3, 4 AND 13
THE NUMBERS REVERSED ARE 13, 4 AND 3
PUT IN 3 NUMBERS PLEASE.
?
```

HOMEWORK SOLUTIONS:

1. 233

2. 103

3. 1107

4. 36

5. 1110

6. The quotient of 96236 divided by 14 (6874) is greater than the product of 79 and 87 (6873) by 1.

7b)

```
10  PRINT "CALCULATE AND COMPARE"
20  PRINT "PUT IN 3 NUMBERS PLEASE."
30  INPUT L,P,G
40  S = L+P
50  IF S<G THEN GOTO  100
60  PRINT L;",";P;"AND";G;"ARE THE NUMBERS."
70  PR = L*P*G
80  PRINT "THEIR PRODUCT IS";PR
90  GOTO 20
100 PRINT "THE NUMBERS ARE";L;",";P;"AND";G
110 PRINT "THE NUMBERS REVERSED ARE";G;",";P;"AND";L
120 GOTO 20
130 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

Lesson: 44

AIM: How do we incorporate a flowchart into a computer solution to a problem?

(The material in this lesson can be developed in one full period. To actually have the students run the program(s) will require a session in the computer laboratory.)

PERFORMANCE OBJECTIVES: The student will be able to...

list and use the five component parts of a computer solution in coding a program to solve a problem.

VOCABULARY: arithmetic mean, Fahrenheit degrees

MOTIVATION:

For the problem below, complete each of the following instructions.

1) Write an analysis which includes the development of an algorithm.
2) Develop a plan showing how you use the algorithm to solve the problem.
3) Develop a flowchart.
4) Design a planned output.
5) Code a program.

Problem: Enter 3 test grades. Have the computer determine their arithmetic mean. Then if the mean is greater than or equal to 65, have it display PASSING. Otherwise, display FAILING.

DEVELOPMENT:

1. Elicit answers to the following questions.
   a) What algorithm do we use in order to find the mean? Elicit: AM = (G1+G2+G3)/3 where memory cells G1, G2 and G3 store the test grades and AM stores the arithmetic mean.
   b) What would happen if we left out the parentheses? (You may wish to review the order of arithmetic

operations here by illustrating with specific numbers.)

2a. Develop the following plan with the students.

Plan

1) Put in values for the grades: G1, G2, G3.
2) Calculate the mean: AM = (G1+G2+G3)/3.
3) Check: Is AM ≥ 65?
    - a) If yes, print PASSING. Go to End.
    - b) If no, continue.
4) Print FAILING.
5) End.

2b. Ask the following questions as you develop the plan.

i) What information do we need to enter? Say that we will store the test grades in memory cells G1, G2 and G3.

ii) What algorithm do we want the computer to use in order to calculate the mean? Say that we will store the mean in memory cell AM.

iii) What do we want the computer to check about the value stored in memory cell AM?

iv) If the mean is less than sixty-five, what message do we want the computer to display?

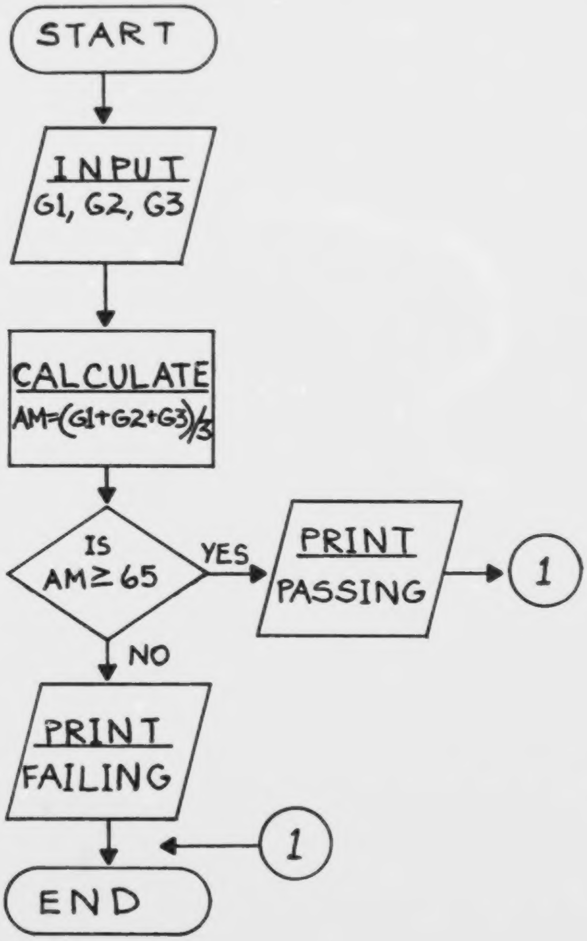
v) What do we want the computer to do after the message FAILING is displayed?

vi) If the mean is greater than or equal to sixty-five, what message do we want the computer to display?

vii) What do we want the computer to do after the message PASSING is displayed?

3a. Ask the students to write the flowchart that follows.

Flowchart


START
INPUT
G1, G2, G3
CALCULATE
AM=(G1+G2+G3)/3
IS
AM≥65
YES
NO
PRINT
PASSING
1
PRINT
FAILING
1
END

3b. Ask the students to trace the flowchart.

4a. Ask the students to design a planned output similar to the one that follows.

Planned Output

```
RUN
AVERAGING TEST GRADES
ENTER 3 TEST GRADES.
?50,70,60
THE MEAN IS 60.
YOU MUST WORK HARDER!
RUN
AVERAGING TEST GRADES
ENTER 3 TEST GRADES.
?80,90,85
THE MEAN IS 85.
YOU PASSED!
```

4b. As they develop the planned output, remind the students of the importance of the title, the instructions to the user, and an explanation of the results.

4c. Point out that the planned output should handle every print statement in the flowchart.

4d. Elicit from the students three grades whose mean is less than 65. Then elicit three grades whose mean is greater than or equal to 65.

5a. Ask the students to code the program as follows.

Program

```
10  PRINT "AVERAGING TEST GRADES"
20  PRINT "ENTER 3 TEST GRADES."
30  INPUT G1,G2,G3
40  AM = (G1+G2+G3)/3
50  PRINT "THE MEAN IS";AM;"."
60  IF AM >= 65 THEN GOTO  90
70  PRINT "YOU MUST WORK HARDER!"
80  GOTO  100
90  PRINT "YOU PASSED!"
100 END
```

5b. Ask, "What would happen if we interchanged (switched) lines 50 and 60?"

5c. Remind the students that we leave underlines at the ends of lines 60 and 80 because as we are writing the program, we do not know the line numbers yet. We do not type the underline, however.

6. If computers are available, ask the students to type in and run the program.

APPLICATIONS:

1. Revise the flowchart developed in this lesson and then code a program that will produce the following output.

```
THE ARITHMETIC MEAN
WHAT WERE THE TEST GRADES?
?50,60,70
50          TEST #1
60          TEST #2
70          TEST #3
60          MEAN
FAILING.
WHAT WERE THE TEST GRADES?
?80,90,85
80          TEST #1
90          TEST #2
85          TEST #3
85          MEAN
PASSING.
WHAT WERE THE TEST GRADES?
?
```

2. For the problem below, complete the following instructions.
    a) Write an analysis which leads to the development of an algorithm.
    b) Write a plan showing how you use the algorithm to solve the problem.
    c) Develop a flowchart.
    d) Design a planned output.
    e) Code a program.

Problem: Have the computer find the mean temperature for the weekend (in Fahrenheit degrees). If the mean temperature is less than 75°, have it display NOT HOT. Otherwise, display HOT. (Use two temperatures, one for Saturday and one for Sunday.)

SUMMARY:

What are the five steps in preparing a complete computer solution to a problem?

HOMEWORK:

1. Bob received grades of 92, 84 and 70 on three tests. Find his test average.

2. The weekly salaries of six employees in a small firm are $240, $245, $245, $250, $250 and $420. When you apply for a job, you are offered $250 or the average salary of the six employees. Which would you choose?

3. For the problem below, complete the following instructions.
   a) Write an analysis which leads to the development of an algorithm.
   b) Write a plan showing how you use the algorithm to solve the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

> Problem: Enter four Fahrenheit temperatures. If their mean is less than or equal to 32°, have the computer display IT IS FREEZING. Otherwise, display IT IS NOT FREEZING.

HOMEWORK SOLUTIONS:

1. 82

2. The average salary, $275, is greater.

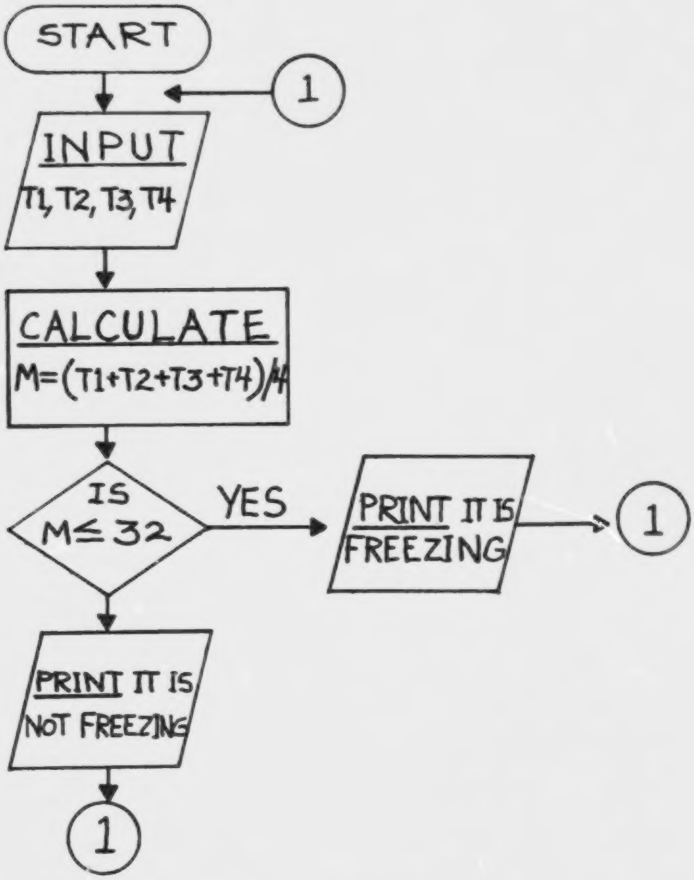
3a) Analysis

M = (T1+T2+T3+T4)/4 where memory cells T1,T2,T3,T4 store the test grades and memory cell M stores the mean.

3b) Plan

1) Put in four Fahrenheit temperatures: T1,T2,T3,T4.
2) Calculate the mean: M = (T1+T2+T3+T4)/4.
3) Check: Is M ≤ 32?
   a) If yes, print IT IS FREEZING. Go to 1.
   b) If no, continue.
4) Print IT IS NOT FREEZING.
5) Go to 1.

3c) Flowchart


START
1
INPUT
T1, T2, T3, T4
CALCULATE
M=(T1+T2+T3+T4)/4
IS M≤32
YES
PRINT IT IS FREEZING
1
PRINT IT IS NOT FREEZING
1

3d) Planned Output

```
HOW COLD IS IT?
ENTER 4 FAHRENHEIT TEMPERATURES.
?40,42,44,46
IT IS NOT FREEZING.
ENTER 4 FAHRENHEIT TEMPERATURES.
?32,29,31,30
IT IS FREEZING.
ENTER 4 FAHRENHEIT TEMPERATURES.
?
```

3e) Program

```
10  PRINT "HOW COLD IS IT?"
20  PRINT "ENTER 4 FAHRENHEIT TEMPERATURES."
30  INPUT T1,T2,T3,T4
40  M = (T1+T2+T3+T4)/4
50  IF M <= 32 THEN GOTO  80
60  PRINT "IT IS NOT FREEZING."
70  GOTO 20
80  PRINT "IT IS FREEZING."
90  GOTO 20
100 END
```

Lesson: 45

AIM: How do we develop a computer solution for problems involving the comparison of areas of triangles?

(The material in this lesson can be developed in one full period. To actually have the students run the program(s) will require a session in the computer laboratory.)

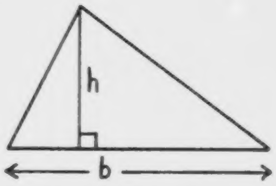
PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate his/her understanding of the formula for finding the areas of triangles by incorporating it into a plan for a computer solution;
2. flowchart a computer solution to a problem;
3. design a planned output;
4. enter, run and correct his/her program.

VOCABULARY: area, triangle, integral value, parallelogram, factors, base, height

MOTIVATION:

1. Find a pair of values for b and h so that the area of the triangle will be greater than thirty square units.



2. Find a pair of values for b and h so that the area of the triangle will be less than thirty square units.
3. Find a pair of values for b and h so that the area of the triangle will be equal to thirty square units.
4. How many integral values are there for b and h for which the area of the triangle will be thirty square units?

DEVELOPMENT:

1. Elicit responses to the motivation problem. Discuss the definition of "integral values" with the students.

2. Present the problem below for a computer solution.

> Problem: Enter the measures of the base and the height of a triangle. Have the computer determine if the area is greater than thirty. (Assume the measures entered are positive.)

3a. Review the algorithm for finding the area of a triangle, given the measures of the base and the height; e.g., $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$. Remind the students to ask themselves, "How do we find the area of a triangle when we know the measures of the base and the height?"

3b. Ask, "To what value will the computed area be compared?" Elicit $A>30$.

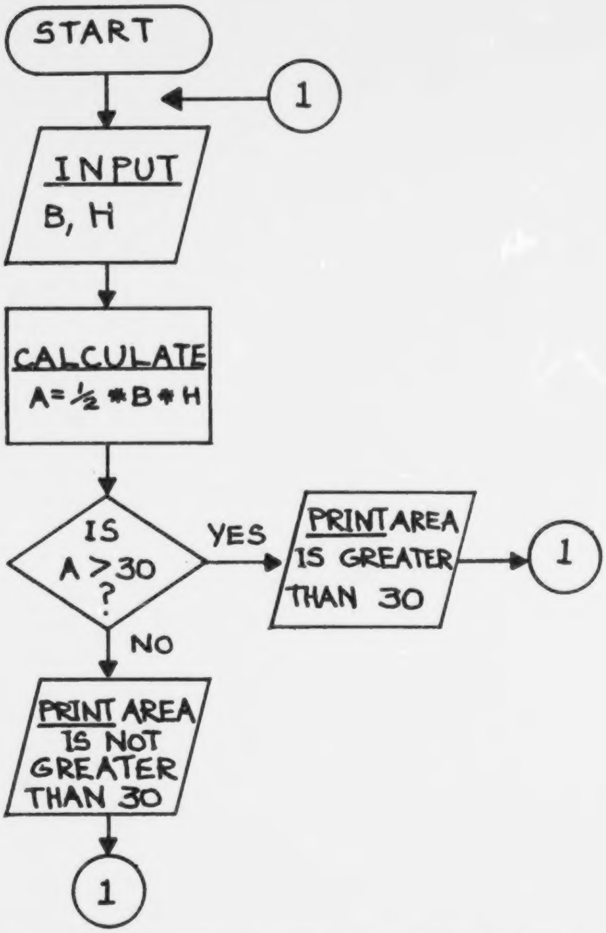
4. Elicit the plan as follows. Remind the students to ask themselves, "How do we use the algorithm to solve the problem?"

Plan:

1) Put in the measures of the base and height of a triangle: B,H.
2) Calculate the area: $A = \frac{1}{2}BH$.
3) Check: Is $A>30$?
    a) If yes, print AREA IS GREATER THAN 30. Go to 1.
    b) If no, continue.
4) Print AREA IS NOT GREATER THAN 30.
5) Go to 1.

5. Ask the students to flowchart the plan as follows.

Flowchart


START
1
INPUT
B, H
CALCULATE
A=½*B*H
IS
A>30
?
YES
PRINT AREA
IS GREATER
THAN 30
1
NO
PRINT AREA
IS NOT
GREATER
THAN 30
1

6. Ask the students to trace the flowchart using the values they determined in the motivation.

7a. Ask the students to design a planned output as follows. (Note to the teacher: Place on the chalkboard for those students who cannot design their own.)

```
COMPARING THE AREA OF A TRIANGLE TO 30
ENTER THE BASE.
?8
ENTER THE HEIGHT.
?3
THE AREA IS 12 SQ. UNITS.
IT IS NOT GREATER THAN 30 SQ. UNITS.
ENTER THE BASE.
?10
ENTER THE HEIGHT.
?7
THE AREA IS 35 SQ. UNITS.
IT IS GREATER THAN 30 SQ. UNITS.
ENTER THE BASE.
?
```

7b. Remind the students to be sure that all parts of the flowchart are handled in the planned output.

7c. Point out the importance of the title, the instructions to the user, and an explanation of the results.

8. Ask the students to code the program, enter it and run it. As the students write the program, stress that when a decision is reached, they are to program the NO before the YES. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you.

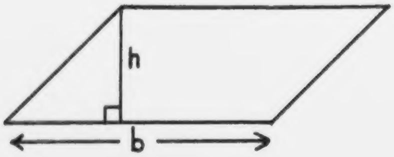
Program:

```
10  PRINT "COMPARING THE AREA OF A TRIANGLE TO 30"
20  PRINT "ENTER THE BASE."
30  INPUT B
40  PRINT "ENTER THE HEIGHT."
50  INPUT H
60  A = 1/2*B*H
70  PRINT "THE AREA IS";A;"SQ. UNITS."
80  IF A>30 THEN GOTO  110
90  PRINT "IT IS NOT GREATER THAN 30 SQ. UNITS."
100 GOTO 20
110 PRINT "IT IS GREATER THAN 30 SQ. UNITS."
120 GOTO 20
130 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

APPLICATIONS:

1.



a) Find a pair of values for b and h so that the area of the parallelogram will be greater than one hundred sq. units.

b) Find a pair of values for b and h so that the area of the parallelogram will be less than one hundred sq. units.

c) Find a pair of values for b and h so that the area of the parallelogram will be equal to one hundred sq. units. How many such integral pairs of values are possible?

2. For the problem below, complete the following instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan showing how you use the algorithm to solve the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

Problem: Enter the measures of the base and the height of a parallelogram. Have the computer determine if the area is not equal to one hundred sq. units.

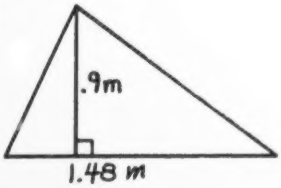
SUMMARY:

What information do we need to be given in order to determine the area of a triangle? ...a parallelogram?

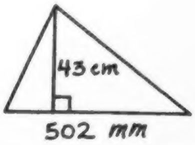
HOMEWORK:

1. Find the area of each.

a)



b)



2. Which polygon has the greater area, a triangle whose base measures 32 mm and whose height measures 10 mm or a parallelogram whose base measures 15 mm and whose height measures 11 mm? How much greater?

3. For the following problem, complete the instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan showing how to use the algorithm to solve the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

Problem: Enter the measures of the base and the height of a triangle. Then enter the measures of the base and the height of a parallelogram. Have the computer determine if the area of the triangle is greater than that of the parallelogram. (Assume all the measures entered are positive.)

HOMEWORK SOLUTIONS:

1. a) 0.666 sq. m or 0.666 m^2
   b) 107930 sq. mm or 1079.3 sq. cm

2. The area of the parallelogram, 165 sq. mm, is greater than the area of the triangle, 160 sq. mm, by 5 sq. mm.

3a) Analysis

For the triangle,

$AT = \frac{1}{2}*BT*HT$ where memory cell BT stores the measure of the base,
memory cell HT stores the measure of the height, and
memory cell AT stores the area.

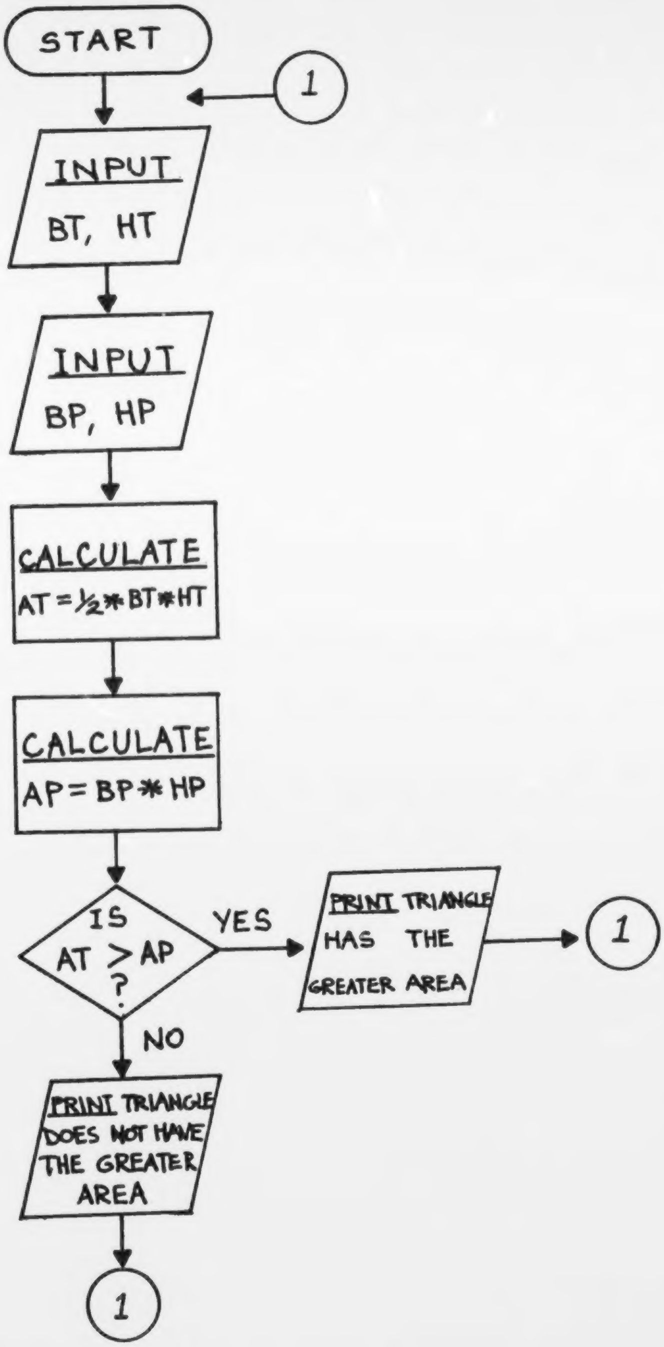
For the parallelogram,

$AP = BP*HP$ where memory cell BP stores the measure of the base,
memory cell HP stores the measure of the height, and
memory cell AP stores the area.

3b) Plan

1) Put in the measures of the base and height of a triangle: BT,HT.
2) Put in the measures of the base and height of a parallelogram: BP,HP.
3) Calculate the area of the triangle: $AT = \frac{1}{2}*BT*HT$.
4) Calculate the area of the parallelogram: $AP = BP*HP$.
5) Check: Is AT > AP?
   a) If yes, print TRIANGLE HAS THE GREATER AREA. Go to 1.
   b) If no, continue.
6) Print TRIANGLE DOES NOT HAVE THE GREATER AREA.
7) Go to 1.

3c) Flowchart


START
1
INPUT
BT, HT
INPUT
BP, HP
CALCULATE
AT = ½ * BT * HT
CALCULATE
AP = BP * HP
IS
AT > AP
?
YES
NO
PRINT TRIANGLE HAS THE GREATER AREA
1
PRINT TRIANGLE DOES NOT HAVE THE GREATER AREA
1

3d) Planned Output

```
COMPARING THE AREAS OF
TRIANGLES AND PARALLELOGRAMS
ENTER THE BASE OF A TRIANGLE.
?10
ENTER THE HEIGHT OF A TRIANGLE.
?6
ENTER THE BASE OF A PARALLELOGRAM.
?7
ENTER THE HEIGHT OF A PARALLELOGRAM.
?5
THE TRIANGLE DOES NOT HAVE THE GREATER AREA.
ENTER THE BASE OF A TRIANGLE.
?8
ENTER THE HEIGHT OF A TRIANGLE.
?6
ENTER THE BASE OF A PARALLELOGRAM.
?10
ENTER THE HEIGHT OF A PARALLELOGRAM.
?2
THE TRIANGLE HAS THE GREATER AREA.
ENTER THE BASE OF A TRIANGLE.
?
```

3e) Program

```
10  PRINT "COMPARING THE AREAS OF"
20  PRINT "TRIANGLES AND PARALLELOGRAMS"
30  PRINT "ENTER THE BASE OF A TRIANGLE."
40  INPUT BT
50  PRINT "ENTER THE HEIGHT OF A TRIANGLE."
60  INPUT HT
70  PRINT "ENTER THE BASE OF A PARALLELOGRAM."
80  INPUT BP
90  PRINT "ENTER THE HEIGHT OF A PARALLELOGRAM."
100 INPUT HP
110 AT = 1/2*BT*HT
120 AP = BP*HP
130 IF AT > AP THEN GOTO  160
140 PRINT "THE TRIANGLE DOES NOT HAVE THE GREATER AREA."
150 GOTO 30
160 PRINT "THE TRIANGLE HAS THE GREATER AREA."
170 GOTO 30
180 END
```

Lesson: 46L

AIM: How do we develop a computer solution for problems involving the measures of complementary angles?

(The material in this lesson can be developed in one full period. To actually have the students run the program(s) will require a session in the computer laboratory.)

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate his/her understanding of finding the measure of the complement of an angle by developing a plan for a computer solution;
2. flowchart a computer solution to the problem;
3. design a planned output;
4. code a program;
5. enter, run and correct his/her program.

VOCABULARY: complement, supplement, double

MOTIVATION:

Where possible, find the measurement of the complement of each angle whose measurement is listed below.

a) 23°
b) 100°
c) 68°
d) 90°
e) 87°

DEVELOPMENT:

1. As you go over the problems in the motivation, ask, "For which measurements are you unable to find the complement? Why?"

2. Present the problem below for computer solution.

Problem: Enter the measure of an angle. Have the computer find and display the measure of the complement of the angle. (Assume the measure entered is positive.)

3a. Review the algorithm for finding the measure of the complement; e.g., C = 90-A, where A represents the measure of an angle and C represents the measure of its complement. Remind the students to ask themselves, "How do we find the measure of the complement of an angle whose measure is known?"

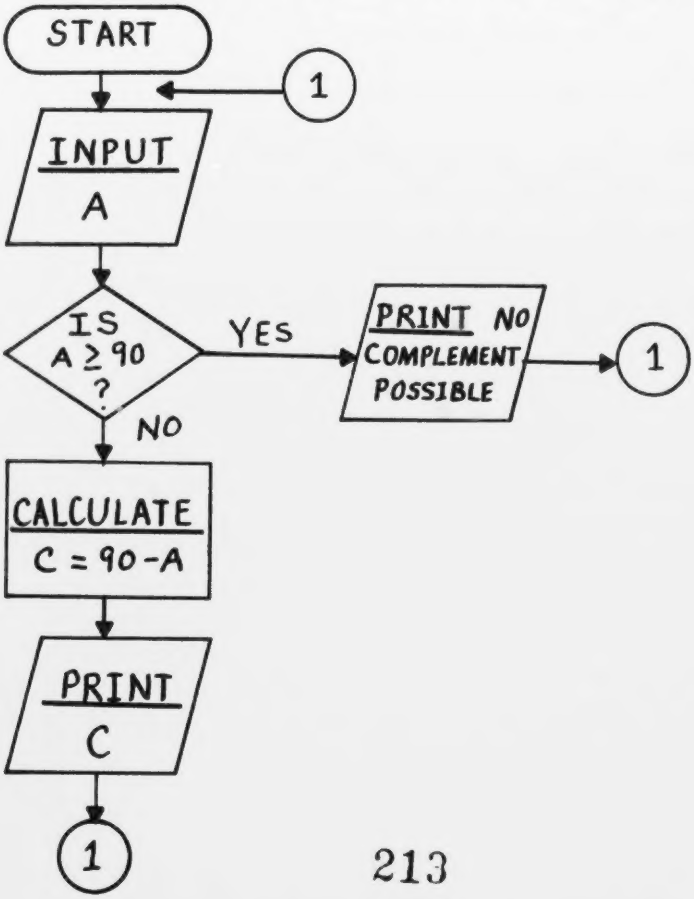
3b. Ask, "For which measures do we not want the computer to find the measure of the complement?" Elicit for $A \geq 90$.

4. Elicit the plan that follows. Remind the students to ask themselves, "How do we use the algorithm to solve the problem?"

Plan:

1) Put in the positive measure of an angle, A.
2) Check: Is $A \geq 90$?
    a) If yes, print NO COMPLEMENT POSSIBLE. Go to 1.
    b) If no, continue.
3) Calculate the measure of the complement: C=90-A.
4) Display C.
5) Go to 1.

5. Ask the students to flowchart the plan as follows.


START
1
INPUT
A
IS
A ≥ 90
?
YES
PRINT NO
COMPLEMENT
POSSIBLE
1
NO
CALCULATE
C = 90-A
PRINT
C
1

6. Ask the students to trace the flowchart using the problems done in the motivation.

7a. Ask the students to design a planned output as follows. (Place on the chalkboard for those students who cannot design their own.)

```
LOOKING FOR COMPLEMENTS
ENTER THE MEASURE OF AN ANGLE.
THE MEASURE MUST BE LESS THAN 90.
?23
THE COMPLEMENT MEASURES 67 DEGREES.
ENTER THE MEASURE OF AN ANGLE.
THE MEASURE MUST BE LESS THAN 90.
?100
NO COMPLEMENT IS POSSIBLE.
THE MEASURE MUST BE LESS THAN 90.
?
```

7b. Remind the students to be sure that all parts of the flowchart are handled in the planned output.

7c. Point out the importance of a title, the instructions to the user, and an explanation of the results.

8. Ask the students to code a program, enter it, and run it. As the students write the program, stress that when a decision is reached, they are to program the NO before the YES. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you.

Program

```
10  PRINT "LOOKING FOR COMPLEMENTS"
20  PRINT "ENTER THE MEASURE OF AN ANGLE."
30  PRINT "THE MEASURE MUST BE LESS THAN 90."
40  INPUT A
50  IF A>=90 THEN GOTO  90
60  C=90-A
70  PRINT "THE COMPLEMENT MEASURES";C;"DEGREES."
80  GOTO 20
90  PRINT "NO COMPLEMENT IS POSSIBLE."
100 GOTO 30
110 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

APPLICATIONS:

1. Where possible, for each angle measurement listed, find

the measure of the supplement of double the angle.

a) 32°
c) 110°
b) 90°

2. For the problem and planned output, complete the following instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Develop a flowchart (plan) showing how you use the algorithm to solve the problem.
   c) Code a program and RUN it on the computer.

Problem: Enter the measure of an angle. Have the computer display the measure of the angle, double the angle and then display the supplement of double the angle. (Assume the angle measure is positive.)

Planned Output

```
THE SUPPLEMENT OF DOUBLE AN ANGLE MEASURE
PUT IN THE MEASURE OF AN ANGLE
THAT IS LESS THAN 90 PLEASE.
?32
ANGLE     DOUBLE     SUPPLEMENT
32        64         116
PUT IN THE MEASURE OF AN ANGLE
THAT IS LESS THAN 90 PLEASE.
?110
PUT IN THE MEASURE OF AN ANGLE
THAT IS LESS THAN 90 PLEASE.
?
```

SUMMARY:

Under what circumstances can the complement of an angle be determined? ...the supplement?

HOMEWORK:

1. a) Where possible, for each angle measurement listed below, find the measure of the complement of one-half of the angle.
      i) 110° ii) 90° iii) 195°
   b) For the problem in "a", under what circumstances will it be impossible to determine the measure of a complement?

2. For the problem below, complete the following instructions.
    - a) Write an analysis which includes the development of an algorithm.
    - b) Write a plan showing how to use the algorithm to solve the problem.
    - c) Develop a flowchart.
    - d) Design a planned output.
    - e) Code a program.

Problem: Enter the measure of an angle. Have the computer display the measure of the angle, one-half of the angle, and the complement of one-half of the angle. (Assume the angle measure entered is positive.)

HOMEWORK SOLUTIONS:

1a) i) 35 ii) 45 iii) not possible

1b) It will be impossible to determine the measure of a complement when the measure of the angle is 180° or greater.

2a) Analysis:

HA = $\frac{1}{2}$A where memory cell A stores the measure of the angle entered and memory cell HA stores the measure of one-half that angle.

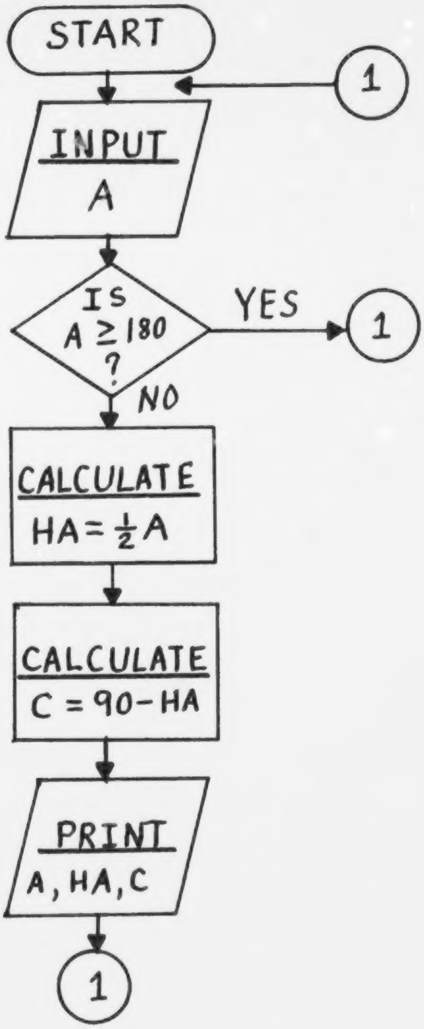
C = 90-HA where memory cell HA stores the measure of one-half the angle entered and memory cell C stores the measure of the complement of one-half that angle.

2b) Plan

1) Put in the positive measure of an angle, A.
2) Check: Is A $\geq$ 180?
    - a) If yes, go to 1.
    - b) If no, continue.
3) Calculate the measure of one-half the angle: HA = $\frac{1}{2}$A.
4) Calculate the measure of the complement of one-half the angle: C = 90 - HA.

5) Display A, HA, C.
6) Go to 1.

2c) Flowchart


START
1
INPUT
A
IS
A ≥ 180
?
YES
1
NO
CALCULATE
HA = ½ A
CALCULATE
C = 90 - HA
PRINT
A, HA, C
1

2d) Planned Output

```
THE COMPLEMENT OF ONE-HALF OF AN ANGLE MEASURE
PUT IN THE MEASURE OF AN ANGLE
THAT IS LESS THAN 180 PLEASE.
?32
ANGLE    ONE-HALF    COMPLEMENT
32       16          74
PUT IN THE MEASURE OF AN ANGLE
THAT IS LESS THAN 180 PLEASE.
?190
PUT IN THE MEASURE OF AN ANGLE
THAT IS LESS THAN 180 PLEASE.
?
```

2e) Program

```
10  PRINT "THE COMPLEMENT OF ONE-HALF OF AN ANGLE MEASURE"
20  PRINT "PUT IN THE MEASURE OF AN ANGLE"
30  PRINT "THAT IS LESS THAN 180 PLEASE."
40  INPUT A
50  IF A > = 180 THEN GOTO 20
60  HA = 1/2*A
70  C = 90 - HA
80  PRINT "ANGLE","ONE-HALF","COMPLEMENT"
90  PRINT A,HA,C
100 GOTO 20
110 END
```

Lesson: 47

AIM: How do we develop a computer solution for problems involving the determination of simple interest?

(The material in this lesson can be developed in one full period. To actually have the students run the program(s) will require a session in the computer laboratory.)

PERFORMANCE OBJECTIVES: The student will be able to...

1. change a number expressed as a percent to a decimal;
2. find a percent of a number;
3. find what percent one number is of another;
4. demonstrate his/her understanding of determining simple interest by developing a plan for a computer solution;
5. flowchart a computer solution to the problem;
6. design a planned output;
7. code a program;
8. enter, run, and correct his/her program.

VOCABULARY: principal, annual rate of interest, interest, percent, decimal

MOTIVATION:

Mr. Sanders needed $900 to improve the kitchen in his home. His uncle, Mr. Reilly, agreed to lend Mr. Sanders the $900 for a period of two years and he would charge him only 4% of the $900 annually. How much of a charge would Mr. Sanders have to pay his uncle after two years?

DEVELOPMENT:

1a. Review with the students that a charge for the use of borrowed money is called interest. The sum of money that Mr. Sanders borrowed, $900, is called the principal. 4% is the annual rate of interest. Two years is the time.

1b. Elicit a procedure for solving the motivation problem; e.g.,

Interest = Principal x Annual Rate (expressed as decimal) x Time (in years)
Interest = $900 x .04 x 2
Interest = $72

If necessary, review the conversion of a percent to a decimal.

1c. Ask, "How much money would Mr. Sanders have to return after two years?" Elicit:

Amount = Principal + Interest
Amount = $900 + $72 = $972

1d. Ask the students to complete Applications 1-4.

2. Present the following problem for a computer solution.

> Problem: Enter the principal and two annual interest rates (both expressed in decimal form). Have the computer determine the annual interest using the lower interest rate. Calculate the interest even if both interest rates are the same.

3. Ask, "How do we solve the problem?" Elicit that we need to have the computer determine which is the lower interest rate and then have it calculate the interest using the algorithm I = PR where I represents the interest, P represents the principal and R represents the annual interest rate. Ask, "Why don't we need to include the time?"

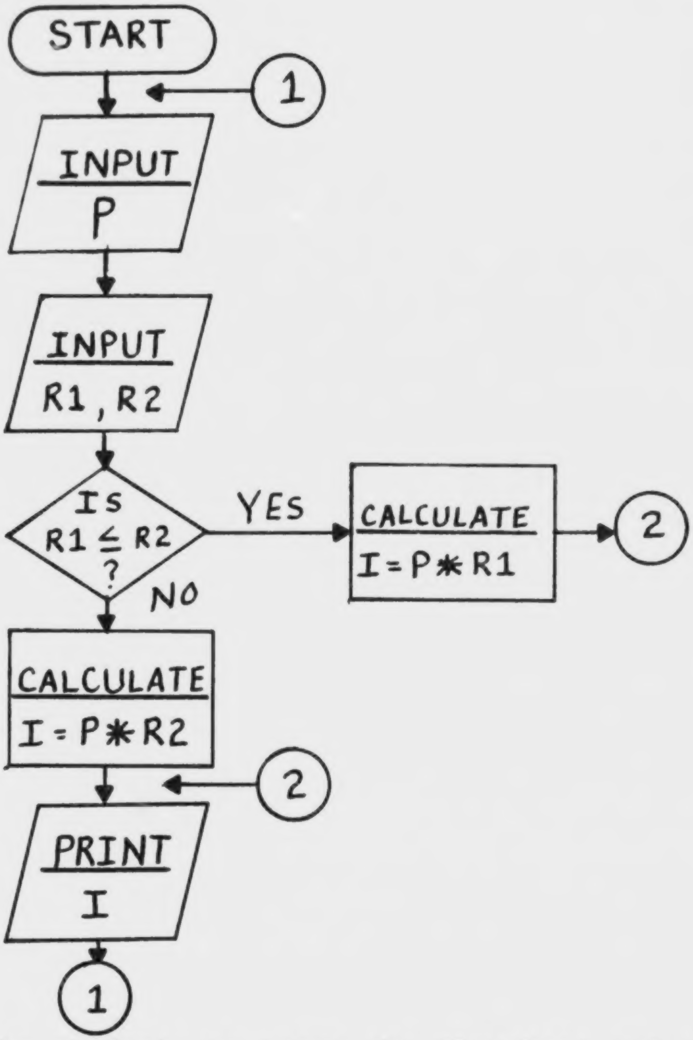
4. Ask the students to incorporate their algorithm into a plan as follows.

Plan

```
1) Put in the principal and the two annual interest
   rates: P, R1, R2.
2) Check: Is R1 ≤ R2?
     a) If yes, calculate: Interest = Principal x Rate 1.
        Then go to 4.
     b) If no, continue.
3) Calculate: Interest = Principal x Rate 2.
4) Display the interest, I.
5) Go to 1.
```

5. Ask the students to flowchart the plan as follows.

Flowchart


START
1
INPUT
P
INPUT
R1, R2
IS
R1 ≤ R2
?
YES
NO
CALCULATE
I = P * R1
2
CALCULATE
I = P * R2
2
PRINT
I
1

6a. Ask the students to trace the flowchart and record their data in a chart as follows.

Principal (P)	Rate 1 (R1)	Rate 2 (R2)	Interest (I)
$1000	.05	.07	$50
$1000	.08	.02	$20

6b. Ask, "What change would there be in the output if the decision is changed to IS R1<R2?"

7a. Ask the students to design a planned output as follows. (Place on the chalkboard for those students who cannot design their own.)

```
GETTING THE LOWER INTEREST RATE
ENTER THE PRINCIPAL.
?1000
ENTER INTEREST RATES 1 AND 2,
IN DECIMAL FORM PLEASE.
?.08,.02
THE RATE IS .02
THE INTEREST ON $1000 AT .02 ANNUALLY
IS $20
ENTER THE PRINCIPAL.
?1000
ENTER INTEREST RATES 1 AND 2,
IN DECIMAL FORM PLEASE.
?.05,.07
THE RATE IS .05
THE INTEREST ON $1000 AT .05 ANNUALLY
IS $50
ENTER THE PRINCIPAL.
?
```

7b. Remind the students to be sure that all parts of the flowchart are handled in the planned output.

7c. Point out the importance of a title, the instructions to the user and an explanation of the results.

8a. Ask the students to code a program, enter it and run it.

8b. As the students write the program, stress that when a decision is reached, they are to program the NO before the YES.

8c. If printers are available, ask each student to submit a listing of his/her program. Otherwise, ask each student to run his/her program for you.

Program

```
10  PRINT "GETTING THE LOWER INTEREST RATE"
20  PRINT "ENTER THE PRINCIPAL."
30  INPUT P
40  PRINT "ENTER INTEREST RATES 1 AND 2,"
50  PRINT "IN DECIMAL FORM PLEASE."
60  INPUT R1,R2
70  IF R1<=R2 THEN GOTO  130
80  I=P*R2
90  PRINT "THE RATE IS";R2
100 PRINT "THE INTEREST ON $";P;"AT";R2;"ANNUALLY"
110 PRINT "IS $";I
120 GOTO 20
130 I=P*R1
140 PRINT "THE RATE IS";R1
150 PRINT "THE INTEREST ON $";P;"AT";R1;"ANNUALLY"
160 GOTO 110
170 END
```

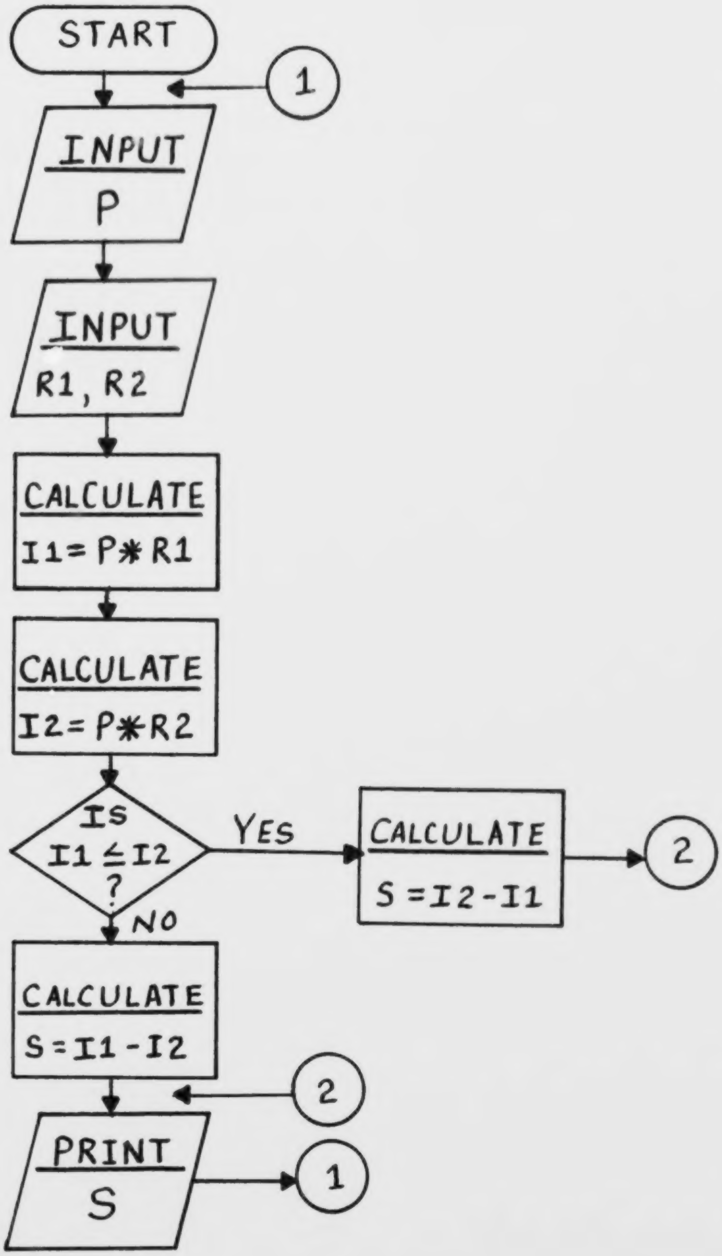
On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

APPLICATIONS:

1. Mr. Boyers borrowed $1400 for 3 years. Find the amount of simple interest he paid if the annual rate of interest was 6%.
2. Miss Carey needed $7500 to buy a car. She borrowed the sum for two years and six months at 8% interest yearly. How much interest did she pay?
3. Mr. Carlson was able to borrow $1000 at one of two annual interest rates, 5% or 7%. How much money would he save by getting the loan at the lower rate?
4. Mr. and Mrs. Stone borrowed $2000 in order to pay for merchandise in cash and get a discount of $100. They borrowed the money for a period of three months at an annual rate of 7%. How much did they gain by making the loan?

5a. Revise the flowchart that was written in today's lesson so that the computer calculates and displays how much the borrower saves by getting the lower interest rate.
5b. Redesign the planned output.
5c. Rewrite the program for the revised problem.
Elicit the flowchart that follows or any other that solves the problem.

Flowchart


START
1
INPUT
P
INPUT
R1, R2
CALCULATE
I1 = P * R1
CALCULATE
I2 = P * R2
IS
I1 ≤ I2
?
YES
NO
CALCULATE
S = I2 - I1
2
CALCULATE
S = I1 - I2
2
PRINT
S
1

Elicit an output similar to the one below.

Planned Output

```
SAVING WITH THE LOWER INTEREST RATE
ENTER THE PRINCIPAL.
?1000
ENTER INTEREST RATES 1 AND 2,
IN DECIMAL FORM PLEASE.
?.08,.02
THE INTEREST ON $1000 AT .08 IS $80
THE INTEREST ON $1000 AT .02 IS $20
THE SAVINGS ARE $60
ENTER THE PRINCIPAL.
?1000
ENTER INTEREST RATES 1 AND 2,
IN DECIMAL FORM PLEASE.
?.05,.07
THE INTEREST ON $1000 AT .05 IS $50
THE INTEREST ON $1000 AT .07 IS $70
THE SAVINGS ARE $20
ENTER THE PRINCIPAL.
?
```

SUMMARY:

1. What information do we need in order to determine the amount of interest a person must pay for the use of money?
2. How do we determine the amount of interest?

HOMEWORK:

1. Write each percent as a decimal.
   a) 35%
   b) 10%
   c) 4%
   d) $8\frac{1}{2}$%
2. Find the simple interest in each.

	Principal	Rate (annual)	Time (in years)
a)	$800	6%	1 year
b)	$725	5%	4 years
c)	$950	8%	6 months

   1 Year = 360 days

3. For the problem which follows, complete the instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan for a computer solution to the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program to solve the problem.

<u>Problem</u>: Enter two principals and an annual interest rate (expressed as a decimal). Have the computer calculate and display the interest on the greater amount of principal. Calculate the interest even if both principals are the same.

HOMEWORK SOLUTIONS:

1. a) .35 c) .04
   b) .10 or .1 d) .085 or $.08\frac{1}{2}$

2. a) $48 b) $145 c) $38

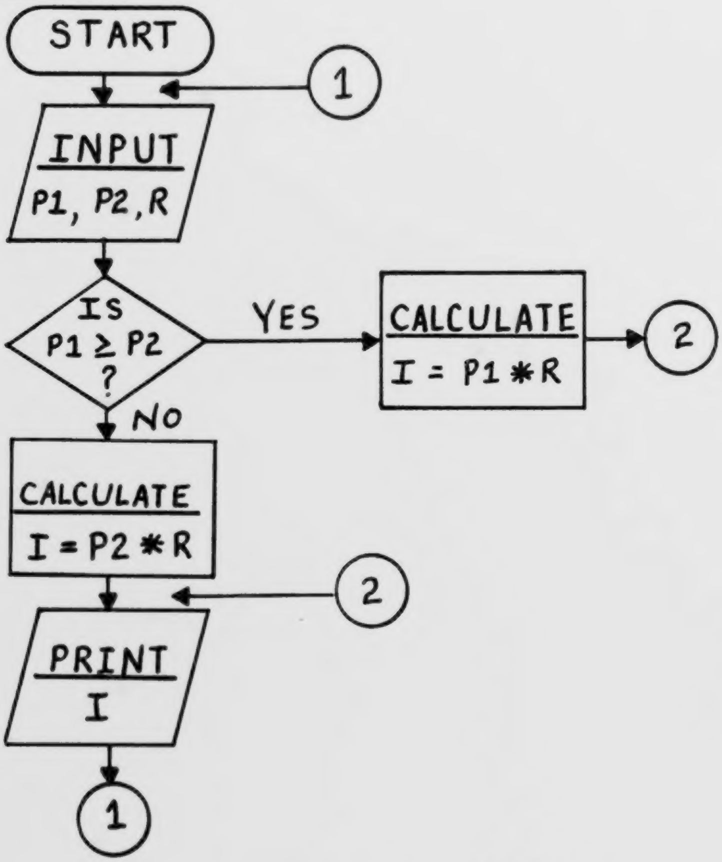
3a) Analysis

Have the computer determine the greater principal and the Interest = Greater Principal x Rate.

3b) Plan

1) Put in the two principals and the annual interest rate: P1,P2,R.
2) Check: Is P1 $\geq$ P2?
   a) If yes, calculate:
      Interest = Principal 1 x Rate. Then go to 4.
   b) If no, continue.
3) Calculate: Interest = Principal 2 x Rate.
4) Display the interest, I.
5) Go to 1.

3c) Flowchart


START
1
INPUT
P1, P2, R
IS P1 ≥ P2 ?
YES
CALCULATE
I = P1 * R
2
NO
CALCULATE
I = P2 * R
2
PRINT
I
1

3d) Planned Output

```
FINDING THE INTEREST ON THE GREATER AMOUNT
TYPE IN PRINCIPALS 1 AND 2.
?800,1000
TYPE IN THE RATE IN DECIMAL FORM.
?.12
THE PRINCIPAL IS $1000
THE INTEREST AT .12 ANNUALLY
IS $120
TYPE IN PRINCIPALS 1 AND 2.
?500,200
TYPE IN THE RATE IN DECIMAL FORM.
?.09
THE PRINCIPAL IS $500
THE INTEREST AT .09 ANNUALLY
IS $45
TYPE IN PRINCIPALS 1 AND 2.
?
```

3e) Program

```
10  PRINT "FINDING THE INTEREST ON THE GREATER AMOUNT"
20  PRINT "TYPE IN PRINCIPALS 1 AND 2."
30  INPUT P1,P2
40  PRINT "TYPE IN THE RATE IN DECIMAL FORM."
50  INPUT R
60  IF P1>=P2 THEN GOTO  120
70  PRINT "THE PRINCIPAL IS $";P2
80  I = P2*R
90  PRINT "THE INTEREST AT";R;"ANNUALLY"
100 PRINT "IS $";I
110 GOTO 20
120 PRINT "THE PRINCIPAL IS $";P1
130 I = P1*R
140 GOTO 90
150 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

Lesson: 48L

AIM: How do we develop a computer solution for problems involving the comparison of numbers written in nondecimal forms?

(The material in this lesson can be developed in one full period. To actually have the students run the program(s) will require a session in the computer laboratory.)

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate his/her understanding of converting numbers expressed as nondecimal base numerals to base ten by developing a plan for a computer solution;
2. develop a flowchart (involving two decisions) to solve a problem;
3. design a planned output;
4. code a program;
5. enter, run and correct his/her program.

VOCABULARY: digit, numeral, decimal numeral, nondecimal numeral, place value, value, number

MOTIVATION:

1. Which represents the larger number,

   43_{five} or 31_{eight}?
2. How much larger?

DEVELOPMENT:

1. a) Elicit the responses to the motivation.
   b) Ask the students to complete Applications 1-3.
2. Present the following problem for a computer solution.

Problem: Enter a base number, N, and the two digits of a base N numeral, one digit at a time. The left digit should be entered first. Enter a second base number, M, and the two digits of a base M numeral, one digit at a time. The left digit should be entered first. Have the computer determine which names the larger number. (Assume N and M are both numbers between 1 and 10.)

3a. Discuss with the students that in order to be able to make a comparison, we must convert both numerals to base ten as we did in our previous problems.

Ask, "If the two-digit numeral is AB_N, the number T, written as a base ten numeral, is obtained by multiplying the digit A by what number? ...by multiplying the digit B by what number?"

Ask, "What do we then do with the products?" Elicit the algorithm T = NA + 1B.

Ask, "Is the multiplication by 1 really necessary?" Explain.

3b. Ask, "Similarly, if the two-digit numeral is CD_M, the number R, written as a base ten numeral, is obtained using which algorithm?" Elicit R = MC + D.

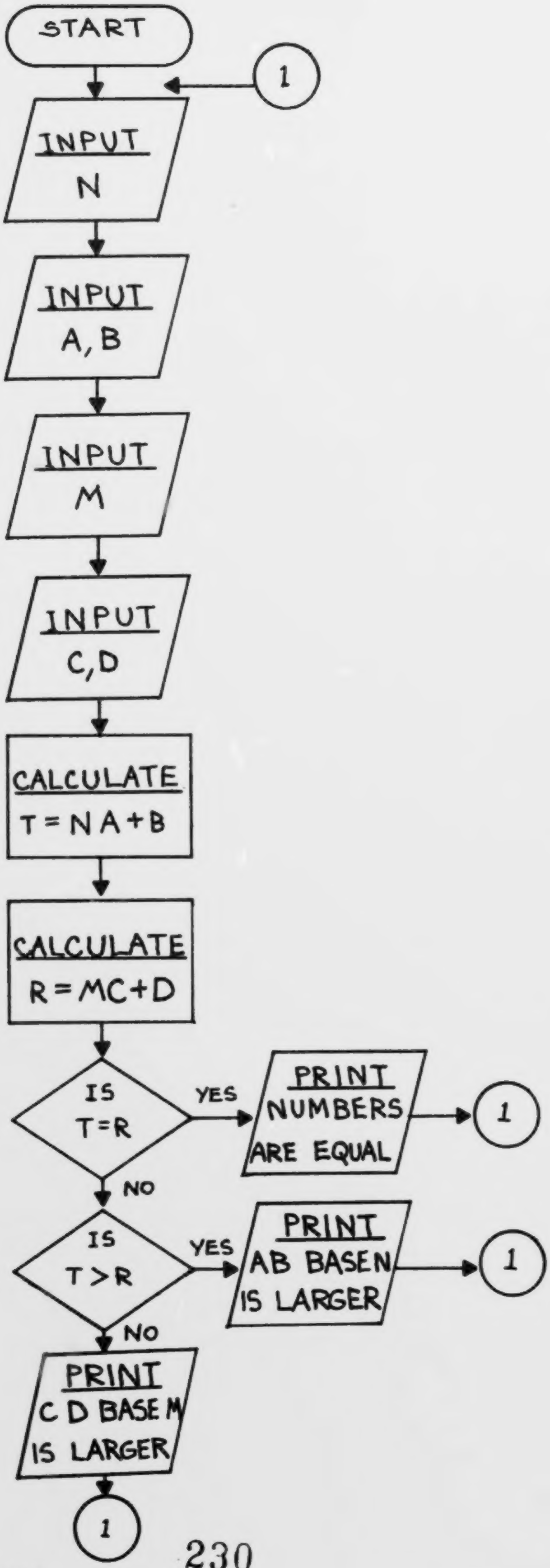
4. Elicit the plan below from the students. Remind the students to ask themselves, "How do we use the algorithm to solve the problem?"

Plan

1) Put in a base number: N.
2) Put in the digits of the base N numeral: A,B.
3) Put in a second base number: M.
4) Put in the digits of the base M numeral: C,D.
5) Calculate: T = NA + B.
6) Calculate: R = MC + D.
7) Check: Is T=R?
    a) If yes, print NUMBERS ARE EQUAL. Go to 1.
    b) If no, continue.
8) Check: Is T>R?
    a) If yes, print A B BASE N IS LARGER. Go to 1.
    b) If no, continue.
9) Print C D BASE M IS LARGER.
10) Go to 1.

5. Ask the students to flowchart the plan as follows.

Flowchart


START
1
INPUT
N
INPUT
A, B
INPUT
M
INPUT
C,D
CALCULATE
T = N A + B
CALCULATE
R = MC + D
IS
T=R
YES
PRINT
NUMBERS
ARE EQUAL
1
NO
IS
T > R
YES
PRINT
AB BASE N
IS LARGER
1
NO
PRINT
C D BASE M
IS LARGER
1

6. Ask the students to trace the flowchart using the problems done earlier (from the Applications).

7a. Ask the students to design a planned output as follows. (Place on the chalkboard for students who cannot design their own.)

```
COMPARING NUMBERS WRITTEN AS NONDECIMAL NUMERALS
ENTER A BASE NUMBER.
?5
ENTER A NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?4,3
ENTER ANOTHER BASE NUMBER.
?8
ENTER ANOTHER NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?3,1
3   1   BASE 8 IS LARGER.
ENTER A BASE NUMBER.
?5
ENTER A NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?1,2
ENTER ANOTHER BASE NUMBER.
?3
ENTER ANOTHER NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?2,1
1   2   BASE 5 = 2   1   BASE 3
ENTER A BASE NUMBER.
?6
ENTER A NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?2,4
ENTER ANOTHER BASE NUMBER.
?4
ENTER ANOTHER NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?3,2
2   4   BASE 6 IS LARGER.
ENTER A BASE NUMBER.
?
```

7b. Remind the students to be sure that all parts of the flowchart are handled in the planned output.

7c. Point out the importance of the title, instructions to the user and an explanation of the results.

8. Ask the students to code the program, enter it, and run it. As the students write the program, stress that when a decision is reached, they are to program the NO before the YES. If printers are available, ask each student to submit a listing of his/her program. Otherwise, each student is to run his/her program for you.

Program

```
10  PRINT "COMPARING NUMBERS WRITTEN AS NONDECIMAL NUMERALS"
20  PRINT "ENTER A BASE NUMBER."
30  INPUT N
40  PRINT "ENTER A NUMERAL WITH 2 DIGITS."
50  PRINT "BEGIN WITH THE LEFT DIGIT."
60  PRINT "ONE DIGIT AT A TIME PLEASE!"
70  INPUT A,B
80  PRINT "ENTER ANOTHER BASE NUMBER."
90  INPUT M
100 PRINT "ENTER ANOTHER NUMERAL WITH 2 DIGITS."
110 PRINT "BEGIN WITH THE LEFT DIGIT."
120 PRINT "ONE DIGIT AT A TIME PLEASE!"
130 INPUT C,D
140 T=N*A+B
150 R=M*C+D
160 IF T=R THEN GOTO  200
170 IF T>R THEN GOTO  220
180 PRINT C;D;"BASE";M;"IS LARGER."
190 GOTO 20
200 PRINT A;B;"BASE";N;"=";C;D;"BASE";M
210 GOTO 20
220 PRINT A;B;"BASE";N;"IS LARGER."
230 GOTO 20
240 END
```

The spacing will vary depending on the computers your students are using. Vary the planned output and program accordingly.

9. Ask the students who complete the program, to consider Application problems 4 and 5.

APPLICATIONS:

1a. Write each as a base ten numeral.

(i) 24_{six} (ii) 78_{nine} (iii) 11_{two}

1b. Arrange the numbers in 1a in descending order.

2. Write true or false for each.

a) $34_{five} = 23_{eight}$ b) $54_{seven} > 47_{nine}$

3. What is the value of "2" in 23_{nine}?

4. Which base ten numeral does each name?

   a) 423_{five} b) 213_{four} c) 324_{six}

5. For the problem, complete the following instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan showing how to use the algorithm to solve the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program and RUN it on the computer.

Problem: Enter the three digits of a number written as a base five numeral, one digit at a time. The left most digit should be entered first. Enter the three digits of a second number written as a base seven numeral, one digit at a time. The left most digit should be entered first. Have the computer determine which number is greater.

SUMMARY:

1. What is the place value of the left digit in a number written as a two-digit base N numeral? ...value...?
2. What is the value of the largest digit that can be used in a number written as a two-digit base N numeral?

HOMEWORK:

1a. Convert each to a base ten numeral.

(i) 43_{five} (ii) 32_{seven} (iii) 65_{eight}

1b. Reverse the digits of each numeral in 1a. and convert each to a base ten numeral.

2. For the problem, complete the following instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan showing how to use the algorithm to solve the problem.

c) Develop a flowchart.
d) Design a planned output.
e) Code a program.

Problem: Enter a base number, N, and the two digits of a number written as a base N numeral, one digit at a time. The left digit should be entered first. Have the computer determine which is larger, the number entered or the number obtained when its digits are reversed.

HOMEWORK SOLUTIONS:

1a. (i) 23 (ii) 23 (iii) 53

1b. (i) $34_{\text{five}} = 19$ (ii) $23_{\text{seven}} = 17$ (iii) $56_{\text{eight}} = 46$

2a) Analysis

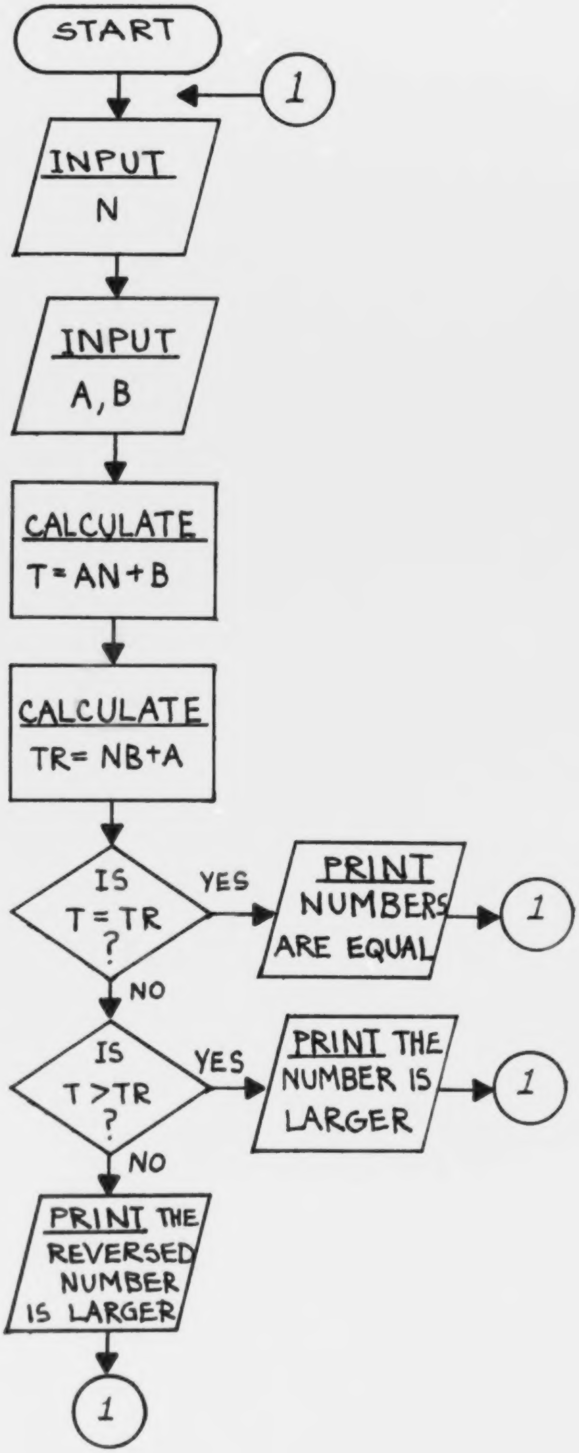
$T = NA + B$ where memory cells A and B store the two digits of the number, memory cell N stores the base number, and memory cell T stores the number written as a base ten numeral.

$TR = NB + A$ where memory cell TR stores the number obtained when the digits are reversed.

2b) Plan

1) Put in a base number: N.
2) Put in the digits of the base N numeral: A,B.
3) Calculate: $T = NA + B$.
4) Calculate: $TR = NB + A$
5) Check: Is $T = TR$?
   a) If yes, print NUMBERS ARE EQUAL. Go to 1.
   b) If no, continue.
6) Check: Is $T>TR$?
   a) If yes, print THE NUMBER IS LARGER. Go to 1.
   b) If no, continue.
7) Print THE NUMBER WITH ITS DIGITS REVERSED IS LARGER. Go to 1.
8) Go to 1.

2c) Flowchart


START
1
INPUT
N
INPUT
A, B
CALCULATE
T = AN + B
CALCULATE
TR = NB + A
IS
T = TR
?
YES
PRINT
NUMBERS
ARE EQUAL
1
NO
IS
T > TR
?
YES
PRINT THE
NUMBER IS
LARGER
1
NO
PRINT THE
REVERSED
NUMBER
IS LARGER
1

2d) Planned Output

```
REVERSING THE DIGITS AND COMPARING
ENTER A BASE NUMBER.
?4
ENTER A NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?1,2
THE 'REVERSED NUMBER' IS  2  1  BASE 4
IT IS LARGER THAN THE NUMBER.
ENTER A BASE NUMBER.
?3
ENTER A NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?2,2
THE 'REVERSED NUMBER' IS  2  2  BASE 3
IT IS EQUAL TO THE NUMBER.
ENTER A BASE NUMBER.
?7
ENTER A NUMERAL WITH 2 DIGITS.
BEGIN WITH THE LEFT DIGIT.
ONE DIGIT AT A TIME PLEASE!
?3,2
THE 'REVERSED NUMBER' IS  2  3  BASE 7
IT IS SMALLER THAN THE NUMBER.
ENTER A BASE NUMBER.
?
```

2e) Program

```
10  PRINT "REVERSING THE DIGITS AND COMPARING"
20  PRINT "ENTER A BASE NUMBER."
30  INPUT N
40  PRINT "ENTER A NUMERAL WITH 2 DIGITS."
50  PRINT "BEGIN WITH THE LEFT DIGIT."
60  PRINT "ONE DIGIT AT A TIME PLEASE!"
70  INPUT A,B
80  T = N*A+B
90  TR = N*B+A
100 PRINT "THE 'REVERSED NUMBER' IS";B;A;"BASE";N
110 IF T = TR THEN GOTO  150
120 IF T > TR THEN GOTO  170
130 PRINT "IT IS LARGER THAN THE NUMBER."
140 GOTO 20
150 PRINT "IT IS EQUAL TO THE NUMBER."
160 GOTO 20
170 PRINT "IT IS SMALLER THAN THE NUMBER."
180 GOTO 20
190 END
```

On some computers, blanks are needed on one or both sides of a displayed number. Adjust the program accordingly.

Lesson: 49

AIM: How do the surface areas of rectangular prisms with a fixed volume compare?

PERFORMANCE OBJECTIVES: The student will be able to...

1. define the concept of volume;
2. identify and use the formula for the volume of a rectangular prism;
3. define the concept of surface area;
4. identify and use the formula for the surface area of a rectangular prism;
5. find integral measures for the dimensions of rectangular prisms with a given fixed integral volume (i.e., find factor triples of natural numbers).
6. find the surface areas of rectangular prisms, given the measures of the dimensions.

VOCABULARY: rectangular prism, cube, factor(s), integral dimensions, surface area, volume, cubic units, total surface area, lateral surface area, faces, bases, edges, vertices

MOTIVATION:

Find dimensions for a rectangular prism whose volume is 64 cubic units.

DEVELOPMENT:

1. a) Review with the students the definition of volume as the measure of the interior of a rectangular prism (or any space figure).
   b) Ask, "What type of unit is used to measure volume?" Elicit that a unit of space or a unit solid is needed. Ask, "What type of unit solid is usually used?" Elicit a cube.
   c) Ask, "What formula is used to determine the volume of a rectangular prism?"

Elicit V = lwh where V represents the volume,
l represents the length,
w represents the width, and
h represents the height.

Stress that length, width, and height must be expressed in the same unit of measure.

d) Ask the students to complete Applications 1-3.

2. Elicit responses to the motivation problem.
Ask, "How can we systematically determine integral measures for the dimensions of all possible rectangular prisms whose volume is 64 cubic units?" Elicit some systematic method from the students for determining the factor triples for 64; e.g.,

Length	Width	Height
1	1	64
1	2	32
1	4	16
1	8	8
2	2	16
2	4	8
4	1	16
4	2	8
4	4	4
8	1	8
8	2	4
16	1	4
16	2	2
32	1	2
64	1	1

Beginning with 1, ask, "What is the cofactor of 1?" Ask the students to list all the possible factor pairs of 64. Then ask, "What is the cofactor of 2?" Ask the students to list all possible factor pairs of 32... and so on until all the factors of 64 have been considered. Then ask the students to eliminate all repetitions.
Encourage the students to come up with shortcuts. Ask, "Can you predict when a repetition will occur?"

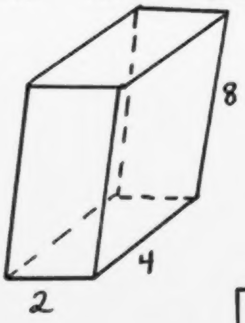
3. Say that now suppose we want to find the surface area of each prism whose dimensions are listed in our final

table; e.g.,

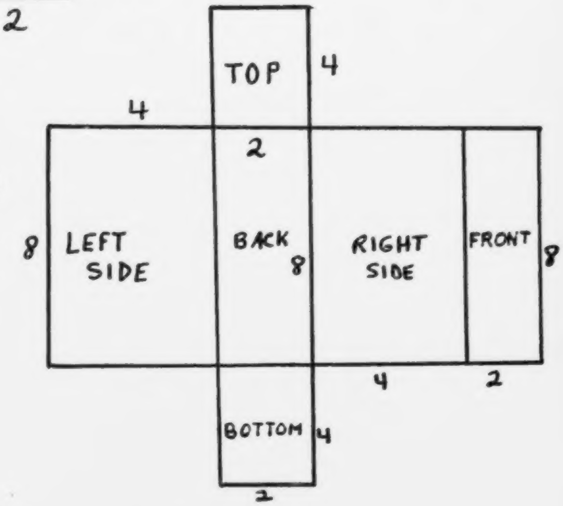
Length	Width	Height
1	1	64
1	2	32
1	4	16
1	8	8
2	2	16
2	4	8
4	4	4

3. a) Ask, "What do we mean by the surface area of a rectangular prism?" Elicit that the total surface area is equal to the sum of the areas of all six faces of the rectangular prism while the lateral surface area excludes the areas of the bases. You might wish to review the definitions of "face," "base," "edge," and "vertex" here. (See Application 4.)

b) Ask the students to examine the prism whose dimensions are 2 by 4 by 8 first.



Ask the students to imagine opening up and flattening out the surface of the prism as follows. (You might wish to make a model to use here.)



Area of the top = length x width = 2X4 = 8
Area of the bottom = length x width = 2X4 = 8
Area of the front = length x height = 2X8 = 16
Area of the back = length x height = 2X8 = 16
Area of the left side = width x height = 4x8 = 32
Area of the right side = width x height = 4x8 = 32
Total Surface Area = 112 sq. units

c) Ask, "How many different areas are there? How are they obtained?"

Elicit: Area of the top or bottom $= lw$
Area of the front or back $= lh$
Area of either side $= wh$

d) Ask, "Once we have determined these areas, how can we then determine the total surface area?" Elicit $SA = 2\ (lw + lh + wh)$ or $SA = 2lw + 2lh + 2wh$. Ask, "What property of mathematics is illustrated by $2(lw+lh+wh) = 2lw + 2lh + 2wh$?"

e) Ask the students to use one of the two formulas to determine the total surface area of each prism the measures of whose dimensions are listed in the table; i.e.,

Dimensions	Surface Areas
1,1,64	258
1,2,32	196
1,4,16	168
1,8,8	160
2,4,8	112
4,4,4	96

f) Ask, "Which rectangular prism has the smallest surface area? What special name is given to a rectangular prism whose dimensions all have the same length?"

4. Ask the students to complete Application 5.

APPLICATIONS:

1. How many cubic inches are there in a box 14 inches long, 8 inches wide, and 2 inches deep?

2. A box is 1 yard long, 1.5 feet wide, and 9 inches deep. What is the volume of the box?

3. A box contains 144 cubic feet. Find its height if it is 9 feet long and 4 feet wide.

4. a) Complete the following table.

Name of a space figure	Number of faces	Number of vertices	Number of edges
Rectangular prism	6	8	12
Triangular prism			
Hexagonal prism			
Square pyramid			
Pentagonal pyramid			

b) What is the relationship between the number of faces, vertices and edges in the prisms and pyramids listed in a? (Elicit Euler's formula $F + V - E = 2$.)

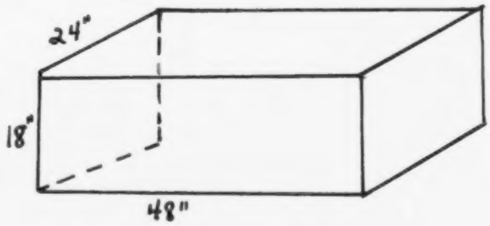
5. Develop a chart showing the surface areas of all rectangular prisms each of whose dimensions has a measure that is an integer and whose volume is 56 cu. units.

SUMMARY:

1. How do we find the volume of a rectangular prism?
2. How do we find the total surface area of a rectangular prism?

HOMEWORK:

1. A room is 15 ft. long, 12 ft. wide, and 8 ft. high. Find the number of square feet of wall paper that would be needed to paper the four walls.
2. An open-top aquarium in the form of a rectangular prism is made of plate glass. If the dimensions of the tank are 24 inches by 48 inches by 18 inches high, find the number of square feet of plate glass required to make the tank.



3. a) Develop a chart showing the total surface areas of all rectangular prisms each of whose dimensions has a measure that is an integer and whose volume is 216 cc.
   b) Which prism has the smallest surface area?

HOMEWORK SOLUTIONS:

1. 432 sq. ft.
2. 26 sq. ft.

3a)

Dimensions (in cm)			Surface Areas (in cc)
1	1	216	866
1	2	108	652
1	3	72	582
1	4	54	548
1	6	36	516
1	8	27	502
1	9	24	498
1	12	18	492
2	2	54	440
2	3	36	372
2	4	27	340
2	6	18	312
2	9	12	300
3	3	24	306
3	4	18	276
3	6	12	252
3	8	9	246
4	6	9	228
6	6	6	216

3b) The cube (6,6,6) has the smallest surface area.

Lesson: 50

AIM: How do we use a READ-DATA statement in a program?

PERFORMANCE OBJECTIVES: The student will be able to...

1. incorporate a READ-DATA statement into a plan (and flowchart) for a computer solution to a problem;
2. predict the output in programs where the READ-DATA statement is used.

VOCABULARY: READ-DATA statement, dimensions, surface area

MOTIVATION:

Suppose we want to write a program that will produce the chart below:

```
FINDING THE SURFACE AREAS OF RECTANGULAR PRISMS
DIMENSIONS      SURFACE AREAS
1, 1, 64        258
1, 2, 32        196
1, 4, 16        168
1, 8, 8         160
2, 2, 16        136
2, 4, 8         112
4, 4, 4         96
```

What makes this output different from any of our previous outputs?

DEVELOPMENT:

1. In going over the motivation problem, elicit that there are no question marks in the chart. Ask, "How can the data be entered without the question marks appearing in the output?"
2. Say that a different method is used to enter the data; it is called a READ-DATA statement. This statement tells the computer to read a number from a DATA statement and store that number in the memory cell named in the READ statement. For example, let us trace three programs to see how the READ-DATA statement is used.

2a. Program #1

Screen

```
10 READ D
20 PRINT D
30 GOTO 10
40 DATA 8,9,10
50 END
RUN
8
9
10
OUT OF DATA
```

Memory

D
~~8~~
~~9~~
10

i) Discuss with the students that each number from the DATA statement is stored in memory cell D. Line 20 displays all the numbers. Ask the students to notice that there are no question marks in the output.

ii) Point out to the students that after all the data is read, the OUT OF DATA message appears since, when the computer is sent back to line 10 to read a fourth number, there is no more data.
(Note to the teacher: The wording of the message OUT OF DATA is different on each computer. Check your manual.)

iii) Ask the students to notice that the numbers in the DATA statement are separated by commas.

2b. Program #2

Screen

```
10 DATA 1,3,5,7
20 READ E, F
30 PRINT E; F
40 GOTO 20
50 END
RUN
1 3
5 7
OUT OF DATA
```

Memory

E	F
~~1~~	~~3~~
5	7

i) Point out to the students that the DATA statement can appear anywhere in the program.

ii) Ask the students to notice that in line 20 the memory cell names (variables) in the READ statement are separated by a comma.

iii) You may wish to review the use of the semicolon with a PRINT statement (line 30).

2c. Program #3

Screen

```
10 READ R
20 READ S,T
30 DATA 2,4,6,8,10,12
40 PRINT R,S,T
50 GOTO 10
60 DATA 14,16,18
70 END
RUN
2         4         6
8         10        12
14        16        18
OUT OF DATA
```

Memory

R	S	T
~~2~~	~~4~~	~~6~~
~~8~~	~~10~~	~~12~~
14	16	18

i) Point out to the students that even though there are two READ statements, all the numbers from the first DATA statement are read first; then those from the next DATA statement are read.

ii) You may wish to review the use of the comma with a PRINT statement (line 40).

3. Ask the students to complete Applications 1-4. As you go over each trace, ask, "Under what circumstances does an OUT OF DATA message not appear?"

4. Ask the students to look once again at the chart of dimensions and surface areas. (motivation problem) Elicit the analysis below.

Analysis

a) Ask, "What instruction in BASIC gives us the ?'s in our programs?" Say that since there are no question marks, we will not use the INPUT statement in our program. Instead, we will use the READ-DATA statement to enter the data.

b) Ask, "What algorithm will we have the computer use to calculate each surface area?"

5a. Develop the plan below with the students.

Plan

1) Read in values for the dimensions (length, width, and height): L, W, H.
2) Calculate the surface area: SA=2(LW+LH+WH).
3) Print L,W,H,SA.

4) Go to 1.
5) Data 1,1,64,1,2,32,1,4,16.
6) Data 1,8,8,2,2,16,2,4,8,4,4,4

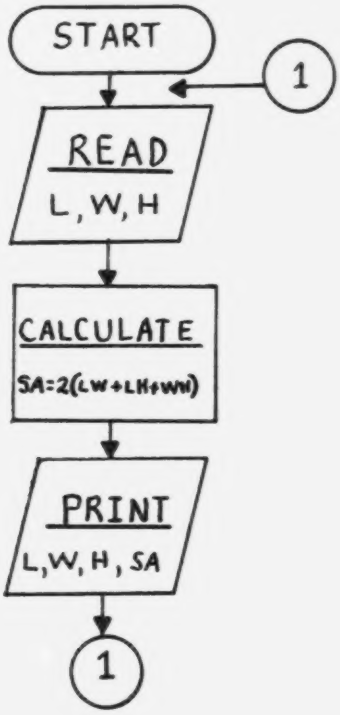
Say that the data is written as length, width, height, length, width, height... This is because the computer will read the values in that order.

5b. Ask the following questions as you develop the plan.

- i) What values do we want to read into the computer? Say that we will store the measures for the length, width, and height in memory cells L, W, and H.
- ii) What algorithm will the computer use to calculate each surface area?
- iii) What do we want the computer to display?
- iv) Where do we send the computer so that another triple of dimensions can be read?
- v) In what order do we list the data in the DATA statement?

6a. Ask the students to flowchart the plan as follows.

Flowchart



6b. Ask the students to trace the flowchart using the data listed in their plan. Ask, "What will happen after seven triples of numbers are read for the length, width and height?" Elicit that we will get an OUT OF DATA message. Say that in future lessons we will learn how to avoid this message.

6c. Say that tomorrow, in the computer laboratory, the students will have the opportunity to code the program and run it.

APPLICATIONS:

Show the output for each program.

1.
```
10 READ X,Y,Z
20 DATA 10,20,30
30 PRINT X
40 PRINT Y
50 PRINT Z
60 END
```

2.
```
10 DATA 3,2,1
20 READ X
30 PRINT X;
40 GOTO 20
50 END
```

3.
```
10 READ X
20 READ Y
30 DATA 5,10,15,20
40 PRINT X; Y
50 GOTO 10
60 END
```

4.
```
10 READ X,Z
20 PRINT X;
30 X=X+1
40 IF X>Z THEN GOTO 70
50 GOTO 20
60 DATA 10,15
70 END
```

SUMMARY:

What things do we have to remember when using a READ-DATA statement in a program?
Elicit:

1. Memory cell names (variables) used in the READ statement are separated by commas.
2. Numbers used in the DATA statement are separated by commas.
3. The DATA statement can appear anywhere in the program.

4. Each number in a DATA statement is read and stored once in a memory cell named in the READ statemnt. (Note to the teacher: Numbers can be read again with a RESTORE command.)

5. When all the numbers in the first DATA statement are read, the computer looks for another DATA statement. If no other DATA statement is available, an OUT OF DATA message appears.

6. No question marks appear in the display when data is entered using the READ-DATA statement.

HOMEWORK: Show the output for each program.

1.
```
10 READ A
20 DATA 3,5,7
30 B=6*A
40 PRINT A,B
50 GOTO 10
60 END
```

2.
```
10 DATA 5,6,4,3
20 READ B,A
30 C=A+3*B
40 PRINT A,B,C
50 GOTO 20
60 END
```

3.
```
10 PRINT "FINDING THE SUM"
20 READ A,B
30 S=A+B
40 PRINT "1ST # IS"; A
50 PRINT "2ND # IS"; B
60 PRINT "SUM IS"; S
70 DATA 8.6, 7.92, 3, .8
80 GOTO 20
90 END
```

4.
```
10 DATA 4,7
20 READ A,B
30 PRINT A;
40 A=A+1
50 IF A>B THEN GOTO 70
60 GOTO 30
70 END
```

HOMEWORK SOLUTIONS:

1. Screen

```
RUN
3        18
5        30
7        42
OUT OF DATA
```

Memory

A	B
~~3~~	~~6*3=18~~
~~5~~	~~6*5=30~~
7	6*7=42

2. Screen

```
RUN
6     5     21
3     4     15
OUT OF DATA
```

Memory

B	A	C
~~5~~	~~6~~	6+3*~~5=~~ ~~6+~~15=21
4	3	3+3*4 3+12=15

3. Screen

```
RUN
FINDING THE SUM
1ST # IS 8.6
2ND # IS 7.92
SUM IS 16.52
1ST # IS 3
2ND # IS .8
SUM IS 3.8
OUT OF DATA
```

Memory

A	B	S
~~8.6~~	~~7.92~~	~~8.6+7.92=16.52~~
3	.8	3+.8=3.8

4. Screen

```
RUN
4  5  6  7
```

Memory

A	B
~~4~~	7
4+1=~~5~~	
5+1=~~6~~	
6+1=~~7~~	
7+1=8	

Lesson: 51L

AIM: How do we code a program that will have the computer display the surface areas of rectangular prisms with a given volume?

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate his/her understanding of finding the surface area of a rectangular prism by coding a program;
2. use the READ-DATA statement in a BASIC program;
3. enter, run, and correct his/her program;
4. revise his/her program to employ different data.

VOCABULARY: total surface area, lateral surface area, cube, edge

MOTIVATION:

Code a program that will solve the following problem.

Problem: Have the computer print the chart below showing the surface areas of rectangular prisms, each of whose dimensions has a measure that is an integer and whose volume is 64 cu. units.

```
FINDING THE SURFACE AREAS OF RECTANGULAR PRISMS
DIMENSIONS     SURFACE AREAS
1,1,64         258
1,2,32         196
1,4,16         168
1,8,8          160
2,2,16         136
2,4,8          112
4,4,4          96
OUT OF DATA
```

DEVELOPMENT:

1a. Ask the students to use the flowchart prepared in the previous lesson and the planned output given in the statement of the problem to code the program below.

Program

```
10 PRINT "FINDING THE SURFACE AREAS OF RECTANGULAR PRISMS"
20 PRINT "DIMENSIONS", "SURFACE AREAS"
30 READ L,W,H
```

```
40 SA=2*(L*W+L*H+W*H)
50 PRINT L;",";W;",";H,SA
60 GOTO 30
70 DATA 1,1,64,1,2,32,1,4,16
80 DATA 1,8,8,2,2,16,2,4,8,4,4,4
90 END
```

b) Ask, "What effect does interchanging lines 60 and 70 have?"

c) Ask the students to experiment with different placements for the DATA statements.

2. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

3. Circulate around the laboratory assisting those students who need it.

4. Group the students who are unable to write the program. Work through some of the coding with this group.

5. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

1. Revise the program written in today's lesson so that it will solve the following problem.

> Problem: Have the computer print a chart showing the surface areas of rectangular prisms each of whose dimensions has a measure that is an integer and whose volume is 36 cu. mm.

2a. For each problem, complete the following instructions.

- i) Write an analysis for the problem that leads to an algorithm.
- ii) Write a plan that outlines how you use the algorithm to solve the problem.
- iii) Develop a flowchart.
- iv) Design a planned output.
- v) Code the program and RUN it on the computer.

Problem: Have the computer read the measures of the edges of six cubes. Have it then print a chart showing the measure of an edge, the total surface area, and the lateral surface area of each cube. Use the following data: 10,5,7,4,3,1.

2b. What is the ratio of the total surface area to the lateral surface area for all cubes?

SUMMARY:

1. What advantage is there in using a READ-DATA statement in a program?
2. What disadvantage is there in using a READ-DATA statement in a program?

HOMEWORK:

1. In a house, the dimensions of the rooms are as follows: kitchen 12 ft. by 10 ft. by 8 ft., living room 18 ft. by 12 ft. by 8 ft., 2 bedrooms each 13 ft. by 10 ft. by 8 ft. The total space of the other rooms is 800 cu. ft.
   a) Find the total air space, or volume, of the rooms in the house.
   b) What is the weight of the air in the house? (Weight of air = .08 lb. per cu. ft.)
2. For the problem, complete the following instructions.
   a) Write an analysis of the problem that leads to an algorithm.
   b) Write a plan that outlines how you use the algorithm to solve the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code the program.

Problem: Have the computer read the measures of the edges of four cubes. Have it then print a chart showing the measure of an edge, the volume, and the total surface area of each cube. Use the following data: 1.5,2.3,3.7,4.1.

HOMEWORK SOLUTIONS:

1. a) 5568 cu. ft. b) 445.44 lbs.

2a) Analysis

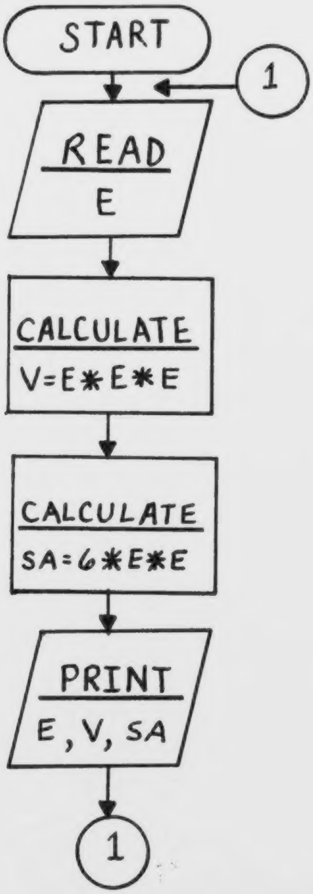
V = E·E·E where memory cell E stores the measure of each edge and
memory cell V stores the volume

SA = 6·E·E where memory cell E stores the measure of each edge and
memory cell SA stores the total surface area.

2b) Plan

```
1)  Read in a value for the measure of each edge, E.
2)  Calculate the volume:  V = E·E·E.
3)  Calculate the surface area:  SA = 6·E·E.
4)  Print E,V,SA.
5)  Go to 1.
6)  Data 1.5,2.3,3.7,4.1
```

2c) Flowchart


START
1
READ
E
CALCULATE
V=E*E*E
CALCULATE
SA=6*E*E
PRINT
E, V, SA
1

2d. Planned Output

```
VOLUMES AND SURFACE AREAS OF CUBES
EDGES       VOLUMES       SURFACE AREAS
(UNITS)     (CU. UNITS)   (SQ. UNITS)
1.5         3.375         13.5
2.3         12.167        31.74
3.7         50.653        82.14
4.1         68.921        100.86
OUT OF DATA
```

2e. Program

```
10  PRINT "VOLUMES AND SURFACE AREAS OF CUBES"
20  PRINT "EDGES","VOLUMES","SURFACE AREAS"
30  PRINT "(UNITS)","(CU. UNITS)","(SQ. UNITS)"
40  READ E
50  V = E*E*E
60  SA = 6*E*E
70  PRINT E,V,SA
80  GOTO 40
90  DATA 1.5,2.3,3.7,4.1
100 END
```

Lesson: 52L

AIM: How do we code a program to have the computer display a table showing the earnings of several employees?

PERFORMANCE OBJECTIVES: The student will be able to...

1. demonstrate his/her understanding of finding commission by developing a plan;
2. flowchart a computer solution to the problem;
3. design a planned output;
4. code a program.

VOCABULARY: commission, salary, rate of commission, sales, hourly rate of pay, wages

MOTIVATION:

The Random Computer Company pays its salespersons a monthly salary of $900 plus 3% commission on monthly sales. Listed below is a chart showing the monthly sales of four salespersons. Compute each salesperson's salary.

Salesperson	Sales	Salary
1	13400	
2	22000	
3	8600	
4	32465	

DEVELOPMENT:

1. Ask the students to fill in the table by computing each salesperson's salary as follows.

Salesperson	Sales	Salary
1	13400	1302
2	22000	1560
3	8600	1158
4	32465	1873.95

(Note to the teacher: You might wish to review the procedure for changing a percent to a decimal and/or the procedure for finding a commission.)

2. Present the following problem for a computer solution.

Problem: Have the computer read the salesperson number and sales for each of four salespersons of the Random Computer Company. Then have it print the table below showing the salesperson number, sales and salary.

SALESPERSON	SALES	SALARY
1	13400	1302
2	22000	1560
3	8600	1158
4	32465	1873.95

3. Elicit the analysis below from the students.

   Analysis

   a) Ask the students to notice that there are no ?'s in the table. Ask, "How can the salesperson numbers and sales be entered without any ?'s appearing in the output?"

   b) Ask, "What algorithm will we have the computer use to calculate the salary for each salesperson?" Elicit SA = .03S+900 where SA represents the salary and S represents the sales.

4. Develop the plan below.

   a) Plan

      1) Read in values for the salesperson number and sales: N,S.
      2) Calculate the salary: SA = .03S+900.
      3) Print N,S,SA.
      4) Go to 1.
      5) Data 1,13400,2,22000,3,8600,4,32465

      Ask the students to notice that the data is written as salesperson number, sales, salesperson number, sales... This is because the computer will read the numbers in that order.

   b) Ask the following questions as you develop the plan.

      i) What values do we want to read into the computer? Say that we will store the salesperson number and sales in memory cells N and S.

ii) What algorithm will the computer use to calculate the salary of each salesperson?

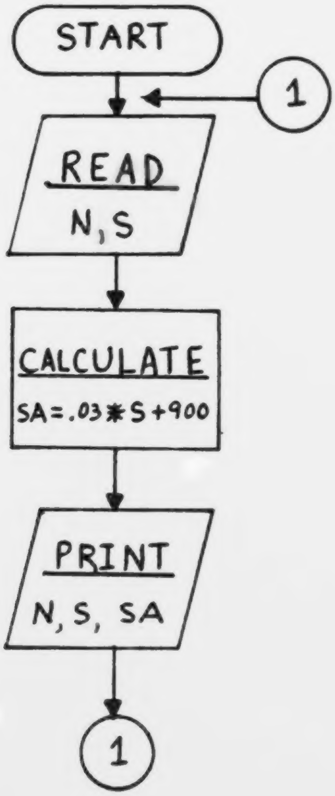
iii) What do we want the computer to display?

iv) Where do we have to send the computer so that other numbers and sales can be read?

v) In what order do we list the data in the DATA statement?

4a. Ask the students to prepare a flowchart as follows:

Flowchart



4b. Ask the students to trace the flowchart using the data from the table given in the statement of the problem.

5a. Ask the students to code the program that follows, recognizing that the planned output is given in the statement of the problem.

Program

```
10 PRINT "SALESPERSON","SALES","SALARY"
20 READ N,S
30 SA=.03*S+900
40 PRINT N,S,SA
50 GOTO 20
60 DATA 1,13400,2,22000,3,8600,4,32465
70 END
```

5b. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

5c. Circulate around the laboratory assisting students where needed.

5d. Group the students who are unable to write the program. Work through some of the coding with this group.

5e. Ask the students who complete the problem presented here, to solve the problem described in the Applications.

APPLICATIONS:

For the problem, complete the following instructions.

a) Write an analysis of the problem that leads to an algorithm.

b) Write a plan that outlines how you use the algorithm to solve the problem.

c) Develop a flowchart.

d) Design a planned output.

e) Code a program and RUN it on the computer.

Problem: An automobile salesman receives 5% on the price of each automobile that he sells. Have the computer print a table showing the car number, the price, and the commission for each automobile sold. Use the following data: 1,12600, 2,15600,3,10800.

SUMMARY:

If Juanita earns $800 per month plus a 5% commission on her sales, how much does she earn at the end of the month?

HOMEWORK:

1. A salesperson receives a commission of 4% on all sales. How much will she receive on sales amounting to $15,000?
2. Find the earnings for...
   a) 7 hours at $4.38 per hour.
   b) 20 hours at $3.90 per hour.
   c) 7.5 hours at $4.80 per hour.
3. Jane works in a store where she earns a salary of $100 a week. She is also given a commission of 4% on her sales. If she sold $1280 worth of merchandise during one week, how much did she earn that week?
4. For the problem below, complete the following instructions.
   a) Write an analysis of the problem that leads to an algorithm.
   b) Write a plan that outlines how you use the algorithm to solve the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

Problem: Have the computer find and display, in chart form, the weekly earnings of four employees at Mike's Diner, each of whom works 40 hours a week. Have it read in the employee number and the hourly rate of pay for each. Use the following data: 1,4.35,2,5.15,3,4.75, 4,4.55.

HOMEWORK SOLUTIONS:

1. $600
2. a) $30.66 b) $78 c) $36
3. $151.20

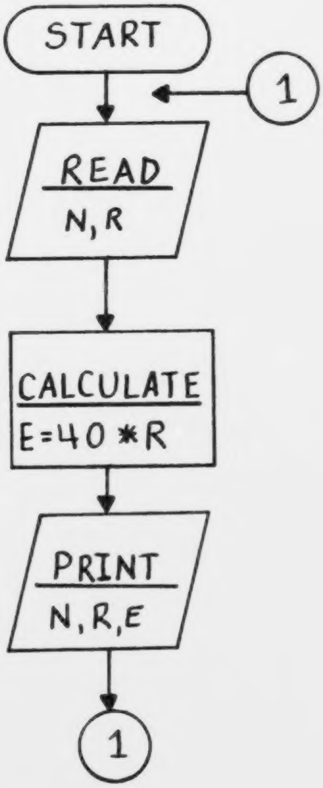
4a) Analysis

E = 40R where memory cell R stores the hourly rate of pay and memory cell E stores the earnings.

4b) Plan

1) Read in values for the employee's number and hourly rate of pay: N,R.
2) Calculate the earnings: E = 40R.
3) Print N,R,E.
4) Go to 1.
5) Data 1,4.35,2,5.15,3,4.75,4,4.55

4c) Flowchart


START
1
READ
N,R
CALCULATE
E=40*R
PRINT
N,R,E
1

4d) Planned Output

```
FINDING EARNINGS
EMPLOYEE #S       RATES OF PAY     EARNINGS
1                 4.35             $174
2                 5.15             $206
3                 4.75             $190
4                 4.55             $182
OUT OF DATA
```

4e) Program

```
10 PRINT "FINDING EARNINGS"
20 PRINT "EMPLOYEE #S","RATES OF PAY","EARNINGS"
30 READ N,R
40 E = 40*R
50 PRINT N,R,E
60 GOTO 30
70 DATA 1,4.35,2,5.15,3,4.75,4,4.55
80 END
```

Lesson: 53

AIM: What is the relationship between the sides and angles of isosceles triangles?

PERFORMANCE OBJECTIVES: The student will be able to...

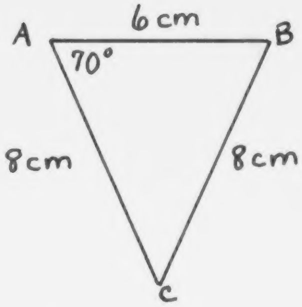
1. define "isosceles triangle";
2. define terms associated with the sides and angles of isosceles triangles (legs, base, base angles, vertex angle);
3. demonstrate his/her understanding of finding the measure of the vertex angle of an isosceles triangle (given the measure of each base angle) by developing a plan (flowchart).

VOCABULARY: isosceles triangle, legs, base, base angles, vertex angle, equilateral triangle

MOTIVATION:

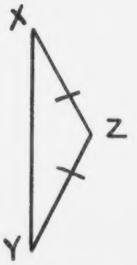
Given: ABC, with $\overline{AB}$ = 6 cm, $\overline{AC}$ = 8 cm
$\overline{BC}$ = 8 cm and m∡A = 70°

1. What is the m∡B?
2. What is the m∡C?



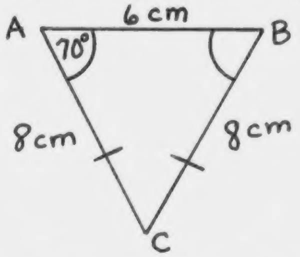
DEVELOPMENT:

1. As you go over the motivation problem, ask the students to notice that ABC has two sides that are equal in length (or congruent). Ask, "What name is given to triangles that have at least two congruent sides? What name is given to each of the congruent sides of isosceles triangles? What name is given to the third side? What name is given to those isosceles triangles that have three congruent sides?" Ask the students to notice that the base need not be at the bottom. Say to the students that when two sides are congruent, this is indicated in the diagram as follows.

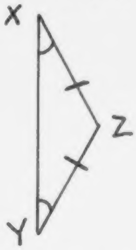


Ask the students to name the legs and base of △XYZ.

2. Remind the students that if two sides of a triangle are congruent, then two angles must also be congruent. Ask, "Where do we find the congruent angles in an isosceles triangle?" Show the students how to mark the angles opposite the congruent sides of ABC as follows.



Ask, "What name is given to the congruent angles of isosceles triangles? What is the m∡B?" Ask the students to notice that the base angles lie "on" the base. Ask, "What name is given to the third angle? Opposite which side of isosceles triangles is the vertex angle located? How do we determine the m∡C?" You might need to review with the students that the sum of the measures of all the angles of a triangle is 180°. Ask the students to name the vertex angle and base angles in XYZ. Mark the diagram as follows.



Ask, "Suppose that the m∡Z = 110°. What is the m∡X? ...m∡Y?"

3. Ask the students to complete Applications 1-5. As you go over question #3, ask the students to notice that all the angle measures listed are less than 90°. Ask, "Is it possible for the measure of each base angle to be greater than or equal to 90°? Why?"

4. Present the following problem for a computer solution.

Problem: Have the computer read the measures of the base angles of five isosceles triangles. Have it then print a chart showing the measure of one base angle and the measure of the vertex angle for each triangle. Use the following data: 80,55,42,22.5,89.9.

5. Elicit the analysis that follows from the students.

Analysis

Ask, "Since we want to have the computer print a chart, how can we enter the measure of each base angle without question marks appearing in the output?" Elicit: Use the READ statement to enter the measures of the base angles. Ask, "What algorithm will we have the computer use in order to determine the measure of the vertex angle?" Elicit: V = 180 - 2B where V represents the measure of the vertex angle and B represents the measure of one base angle.

6a. Develop the plan below with the students.

Plan

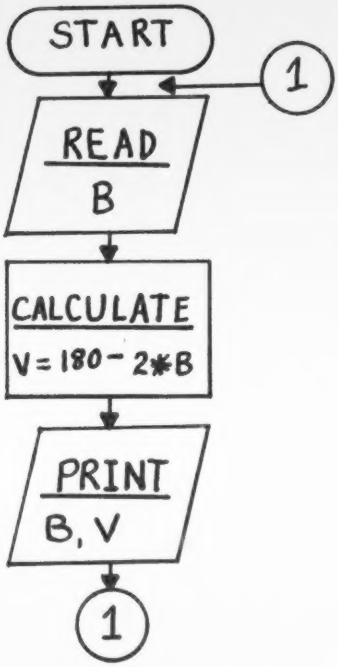
1) Read the measure of each base angle, B.
2) Calculate the measure of the vertex angle: V=180-2B.
3) Print B,V.
4) Data 80,55,42,22.5,89.9
5) Go to 1.

6b. Ask the following questions as you develop the plan.

i) What value do we want to read into the computer for each triangle? Say that we will store the measure of each base in memory cell B.
ii) What algorithm do we want the computer to use in order to calculate the measure of the vertex angle?
iii) What do we want the computer to display?
iv) Where do we have to send the computer so that another measure for the base angle is read?
v) What values do we need to list in our DATA statement?

7a. Ask the students to flowchart a computer solution to the problem as follows:

Flowchart

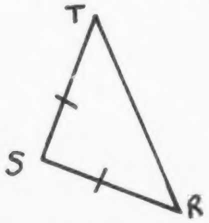

START
1
READ
B
CALCULATE
V=180-2*B
PRINT
B, V
1

7b. Ask the students to trace the flowchart using the data listed in the statement of the problem.

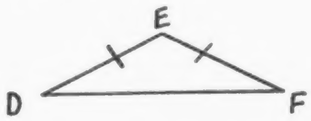
APPLICATIONS:

1. Name the legs, base, vertex angle and base angles in each of the following isosceles triangles.

a)

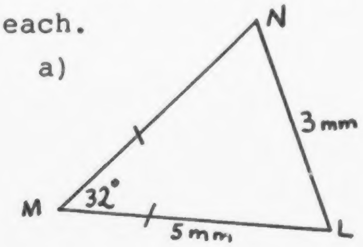


b)

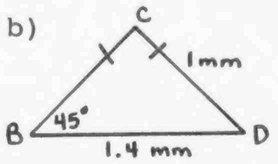


2. Find the measurements of the missing side and angles in each.

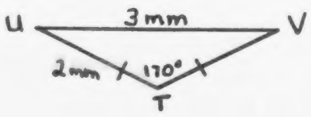
a)


N
3mm
M
32°
5mm
L

b)


C
1mm
B
45°
1.4 mm
D

c)


U
3mm
V
2mm
170°
T

3. Find the measure of the vertex angle of an isosceles triangle if each base angle measures:
   a) 80°   b) 55°   c) 42°   d) 22.5°   e) 89.9°

4. Find the measure of each base angle of an isosceles triangle if the vertex angle measures:
   a) 40°   b) 50°   c) 76°   d) 110°   e) 90°

5. What is the measure of each angle of an equilateral triangle?

SUMMARY:

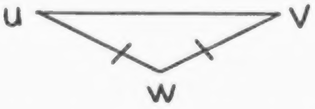
1. If we are given the measure of one base angle of an isosceles triangle, how do we determine the measure of the vertex angle?

2. If we are given the measure of the vertex angle of an isosceles triangle, how do we determine the measure of each base angle?

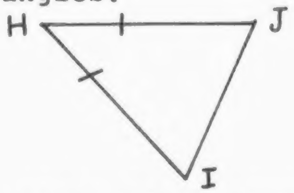
HOMEWORK:

1. Name the legs, base, base angles and vertex angle of each of the following isosceles triangles.

a)



b)

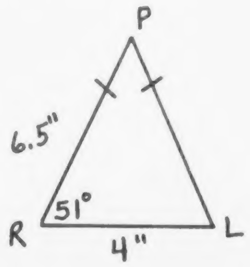


2. Find the measure of the vertex angle of an isosceles triangle if each base angle measures:
   a) 30°   b) 40°   c) 67°   d) 15.5°   e) 83.7°

3. Find the measure of one base angle of an isosceles triangle if the vertex angle measures:
   a) 10°   b) 56°   c) 62°   d) 140°   e) 25°

4a. Find the measurements of the missing side and angles of RLP.

4b. Find the perimeter of RPL.



5. Find the perimeter of an equilateral triangle each of whose sides measures 8.9 mm.

HOMEWORK SOLUTIONS:

1. a) legs: $\overline{UW}$, $\overline{WV}$
   base: $\overline{UV}$
   base angles: $\measuredangle U$, $\measuredangle V$
   vertex angle: $\measuredangle W$

   b) legs: $\overline{HI}$, $\overline{HJ}$
   base: $\overline{IJ}$
   base angles: $\measuredangle I$, $\measuredangle J$
   vertex angle: $\measuredangle H$

2. a) 120° b) 100° c) 46° d) 149° e) 12.6°

3. a) 85° b) 62° c) 59° d) 20° e) 77.5°

4. a) $\overline{PL} = 6.5"$
   $m\measuredangle L = 51°$
   $m\measuredangle P = 78°$

   b) 17"

5. 26.7 mm

Lesson: 54L

AIM: How do we code a program that will have the computer find the measure of the vertex angle of an isosceles triangle when given the measure of one base angle?

PERFORMANCE OBJECTIVES: The student will be able to...

1. design a planned output (in chart form) showing the measure of one base angle and the vertex angle of each isoseles triangle;
2. code a program to have the computer find the measure of the vertex angle of an isosceles triangle, given the measure of one base angle;
3. enter, run, and correct his/her coded program;
4. use a READ-DATA statement in a program.

VOCABULARY: isosceles triangle, legs, base, base angles, vertex angle, READ-DATA statement, equilateral triangle

MOTIVATION:

Design a planned output and then code a program that will solve the following problem.

Problem: Have the computer read the measures of the base angles of five isosceles triangles. Have it then print a chart showing the measure of one base angle and the measure of the vertex angle for each triangle. Use the following data: 80, 55, 42, 22.5, 89.9.

DEVELOPMENT:

1. Ask the students to design a planned output, in chart form, as follows:

Planned Output

```
FINDING THE MEASURE OF THE VERTEX ANGLE
OF AN ISOSCELES TRIANGLE
BASE            VERTEX
ANGLES          ANGLES
80              20
55              70
42              96
22.5            135
89.9            .2
OUT OF DATA
```

2. Then ask the students to use the flowchart developed in the last lesson and the planned output to code a program as follows.

Program

```
10  PRINT "FINDING THE MEASURE OF THE VERTEX ANGLE"
20  PRINT "OF AN ISOSELES TRIANGLE"
30  PRINT "BASE","VERTEX"
40  PRINT "ANGLES","ANGLES"
50  READ B
60  V = 180 - 2*B
70  PRINT B,V
80  GOTO 50
90  DATA 80, 55, 42, 22.5, 89.9
100 END
```

3. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

4. Circulate around the laboratory assisting students as needed.

5. Group the students who are unable to write the program. Work through some of the coding with these students.

6. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

For each of the following problems, complete the instrucitons below.

a) Write an analysis which includes the development of an algorithm.
b) Write a plan for a computer solution to the problem.
c) Develop a flowchart.
d) Design a planned output.
e) Code a program to solve the problem and RUN it on the computer.

1. Problem: Have the computer read the measures of one base angle for each of eight isosceles triangles. Then have the computer print a chart showing the measure of one base angle and the measure of the vertex angle for each triangle, where possible. Use the following data: 8, 82, 18, 118, 53, 90, 77, 25.

2. Problem: Have the computer read the measures of the vertex angle for each of five isosceles triangles. Then have the computer print a chart showing the measure of the vertex angle and one base angle for each triangle. Use the following data: 150, 60, 109, 66.6, 5.5.

SUMMARY:

1. What is the range of possible values for the base angles of isosceles triangles?
2. What is the range of possible values for the vertex angles of isosceles triangels?

HOMEWORK:

1. Find the perimeter of the equilateral triangle each of whose sides measure:
   a) 4" b) 16.5"

2. Find the length of a side of an equilateral triangle whose perimeter is:

   a) 75 cm   b) $19\frac{1}{3}$ yd.

3. Find the perimeter of each of the following isosceles triangles in which "l" represents the length of one leg and "b" represents the length of the base.

   a) l = 8', b = 6'   b) l = 6.2 mm, b = 8.6 mm

4. The perimeter of an isosceles triangle measures 25 cm. If the length of each leg measures 8 cm, find the length of the base.

5. For the following problem, complete the instructions below.

   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan for a computer solution to the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

> Problem: Have the computer read the length (in cm) of the base and the length (in cm) of one leg for each of six isosceles triangles. Have it then print, in chart form, the length of the base, the length of one leg, and the perimeter of each isosceles triangle. Use the following data: 10, 8, 4, 6, 3, 5, 1.4, 1, 3, 2, 4.3, 5.9.

HOMEWORK SOLUTIONS:

1. a) 12"   b) 49.5"
2. a) 25 cm   b) $6\frac{4}{9}$ yd.
3. a) 22'   b) 21 mm
4. 9 cm

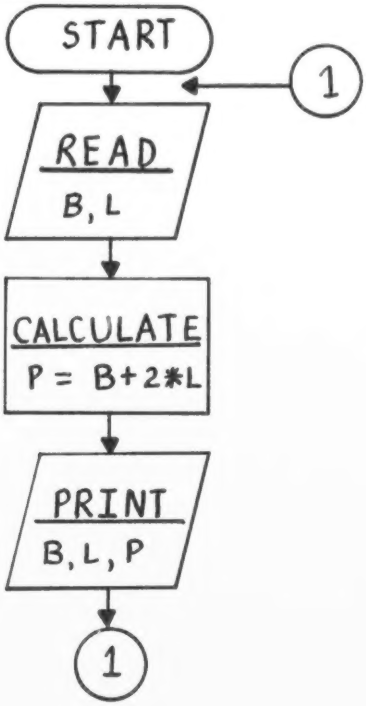
5a) Analysis

P = B+2L where memory cell B stores the length of the base,
memory cell L stores the length of one leg and
memory cell P stores the perimeter.

5b) Plan

1) Read values for the measures of the base and one leg of each triangle: B,L.
2) Calculate the perimeter: P = B+2L.
3) Print B,L,P.
4) Go to 1.
5) Data 10,8,4,6,3,5,1.4,1,3,2,4.3,5.9

5c) Flowchart


START
1
READ
B, L
CALCULATE
P = B+2*L
PRINT
B, L, P
1

5d) Planned Output

```
PERIMETERS OF ISOSCELES TRIANGLES
BASES        LEGS        PERIMETERS
(IN CM)      (IN CM)     (IN CM)
10           8           26
4            6           16
3            5           13
1.4          1           3.4
3            2           7
4.3          5.9         16.1
OUT OF DATA
```

5e) Program

```
10 PRINT "PERIMETERS OF ISOSCELES TRIANGLES"
20 PRINT "BASES","LEGS","PERIMETERS"
30 PRINT "(IN CM)","(IN CM)","(IN CM)"
40 READ B,L
50 P = B + 2*L
60 PRINT B,L,P
70 GOTO 40
80 DATA 10,8,4,6,3,5,1.4,1,3,2,4.3,5.9
90 END
```

Lesson: 55

AIM: How are rate, time and distance related?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the distance, given the rate and time;
2. find the time, given the distance and rate;
3. find the rate, given the distance and time;
4. demonstrate his/her understanding of finding the distance (given the rate and time) by developing a plan (flowchart) for a computer solution;
5. eliminate the OUT OF DATA message by inserting a "flag".

VOCABULARY: rate, time, distance, flag

MOTIVATION:

Lucy drove from Brooklyn, New York to Pine Grove, New Jersey at an average rate of 55 miles per hour (mph) for 2 hours. How far did she drive?

DEVELOPMENT:

1. Elicit a response to the motivation problem. Ask, "What relationship exists among the distance, rate and time?" Elicit D = RT.
2. Ask the students to complete problems 1-3 in the Applications. Stress substitution into the formula D =RT.
3. Pose the following problem: How long will it take Sara to drive 270 miles at the average rate of 45 mph? Say that we will use the formula D = RT. Since the distance is 270 miles and the rate is 45 mph, D = RT

$$270 = 45T.$$

Earlier this term, we solved equations involving multiplication. Ask, "How do we solve for T?" Elicit: Divide both sides by 45, $\frac{270}{45} = \frac{45T}{45}$

$$6 = T \quad \text{or } T = 6.$$

Therefore, Sara needs 6 hours for her trip.

4. Ask the students to complete Applications 4-6. Ask the students to use the formula and solve for T. After discussing problems 4-6, ask the students to generalize. Ask, "How did we obtain the time in each of our problems?" Elicit $T = \frac{D}{R}$.

5. Then pose the following problem:
As a jet plane takes off, the pilot tells the passengers that the trip of 3,000 miles will take 5 hours. What is the average rate at which the plane will be traveling? Say that in this problem we are given the distance, 3,000 miles, and the time, 5 hours. We again use the formula: $D = RT$.
Substitute: $3000 = R(5)$ or $3000 = 5R$
Divide both sides by 5: $\frac{3000}{5} = \frac{5R}{5}$
$600 = R$ or $R = 600$
The jet flies at an average rate of 600 mph.

6. Ask the students to complete Applications 7-9. Again, stress substitution into the formula and solving for R. After discussing problems 7-9, ask the students to generalize. Ask, "How did we obtain the rate in each of our problems?" Elicit $R = \frac{D}{T}$.

7. Ask the students to complete Application question 10, applying the appropriate algorithm in each case.

8. Present the following problem for a computer solution.

<u>Problem</u>: Have the computer read six pairs of values to represent the rates in miles per hour of a car and the times in hours. Have the computer print the chart below.

```
VARYING THE RATE AND TIME
RATES     TIMES     DISTANCES
(MPH)     (HR.)     (MI.)
15        3         45
22        7         154
25        5         125
30        6         180
34        4         136
39        2         78
```

9. Elicit the analysis below from the students.

   Analysis

   a) Ask the students to notice that an OUT OF DATA message does not appear in the chart. Say that in order to avoid the OUT OF DATA message, we will add two more numbers to our DATA statement and then have the computer check for one of them each time a pair of numbers is read. The number that is checked by the computer is called a "flag" because it signals (or flags) the computer that there is no more data. We can use any numbers but usually use numbers that will not be mistaken for the data. Here we will use -99.

   b) Ask, "What algorithm will the computer use to calculate the distance? Elicit D = RT where
      D represents the distance,
      R represents the rate, and
      T represents the time.

10. Develop the plan below with the students.

    a) Plan

       1) Read values for the rate and the time: R, T.
       2) Check: Is R = -99?
          a. If yes, then go to 7.
          b. If no, continue.
       3) Calculate the distance: D = RT.
       4) Print R, T, D.
       5) Go to 1.
       6) Data 15,3,22,7,25,5,30,6,34,4,39,2,-99,-99

       Ask the students to notice that the data is written as rate, time, rate, time,... This is because the computer will read the values in that order. Notice also that two additional values have been annexed.

       7) End.

    b) Ask the following questions as you develop the plan.

       i) What values do we want to read into the computer?

ii) Each time a pair of values is read, what check do we want to make on the value in memory cell R in order to signal (flag) the computer that there is no more data?

iii) If the value in memory cell R is -99, what do we want the computer to do?

iv) If the value in memory cell R is not -99, what algorithm do we want the computer to use in order to calculate the distance?

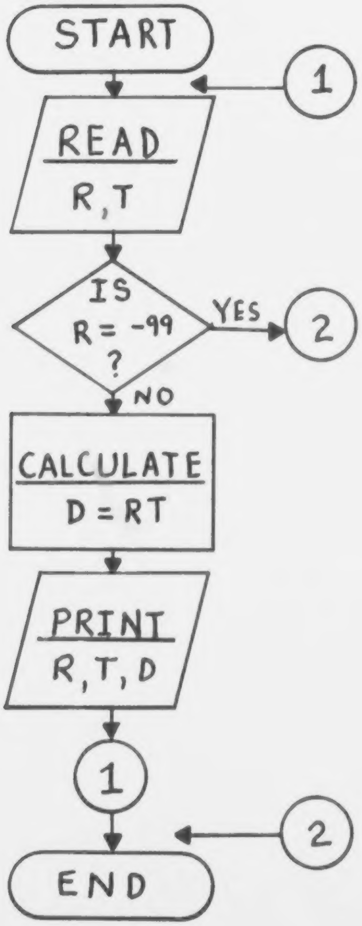
v) What do we want the computer to display?

vi) Where do we have to send the computer so that another rate and time can be read?

vii) In what order do we list the data in the DATA statement?

11. a) Ask the students to develop a flowchart as follows.

Flowchart



b) Ask the students to trace the flowchart using the data in the chart given in the statement of the problem.

c) Ask, "If we are only checking memory cell R, why do we need to put two extra numbers in the DATA statement?" Discuss with the students that when the computer goes back to read a seventh pair of numbers, both memory cell R and T must be filled or there will be an OUT OF DATA message displayed.

APPLICATIONS:

1. How far can Vinny travel in 7 hours at the rate of 35 miles per hour?
2. How far can Al travel in 5 hours at the rate of 48 mph?
3. A train left New York at 11:30 a.m. and arrived in Philadelphia at 1:15 p.m. The train averaged 52 miles per hour. What is the distance from New York to Philadelphia by train?
4. How long will it take Doris to drive 360 miles at a rate of 40 miles per hour?
5. A plane flew 840 miles at an average rate of 160 miles per hour. How long did the trip take?
6. How long will it take a train to travel 300 miles at an average rate of 60 miles per hour?
7. What is the average rate of a plane that travels 2,400 miles in 8 hours?
8. A man drove 180 miles in 5 hours. What was his average rate?
9. A train left New York at 10:00 a.m. and arrived at Washington, D.C. at 1:45 p.m. If the distance from New York to Washington is 225 miles, find the average rate of the train.

10. Fill in the table below by finding the missing measurement.

	Distance (mi.)	Rate (mph)	Time (hr.)
a)	?	25	3
b)	108	36	?
c)	180	?	$4\frac{1}{2}$
d)	24	30	?
e)	36	?	1.5
f)	?	46.2	7

SUMMARY:

1. How do we find the distance, given the rate and time?
2. How do we find the time, given the distance and rate?
3. How do we find the rate, given the distance and time?

HOMEWORK:

1. A plane flew 400 kilometers an hour for $4\frac{3}{4}$ hours. How far did the plane fly?
2. A plane flew 3,600 kilometers at an average rate of 600 kilometers per hour. How long did the trip take?
3. A car traveled 228 miles in 6 hours. What was the average rate?
4. The distance, D, that is covered by a car traveling at an average rate of 40 kilometers per hour for T hours is given by the formula $D = 40T$.
    a) Make a table showing the values of D as T changes from 1 through 10.
    b) What change takes place in the value of D when T...
        i) increases by 1?
        ii) decreases by 1?
        iii) is multiplied by 5?
        iv) is multiplied by 10?
5. Solve and check each equation.
    a) $13 = 2T$
    b) $1.25 = 5X$
    c) $4 = .25Y$
    d) $1.2W = 9.6$
    e) $.02R = 1$
    f) $38 = 9\frac{1}{2}D$

HOMEWORK SOLUTIONS:

1. 1900 km
2. 6 hrs.
3. 38 mph

4a)

T	D
1	40
2	80
3	120
4	160
5	200
6	240
7	280
8	320
9	360
10	400

4b) i) D increases by 40.
ii) D decreases by 40.
iii) D is multiplied by 5.
iv) D is multiplied by 10.

5. a) 6.5 = T
b) 0.25 = X
c) 16 = Y
d) W = 8
e) R = 50
f) 4 = D

Lesson: 56L

AIM: How do we code a program to have the computer print a chart showing the rate, time, and distance?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to have the computer find the distance, given the rate and time;
2. enter, run and correct his/her program;
3. incorporate a "flag" to eliminate the OUT OF DATA message in a program that involves the use of a READ-DATA statement.

VOCABULARY: rate, time, distance, flag, odometer

MOTIVATION:

Code a program that will solve the problem that follows.

Problem: Have the computer read six pairs of values to represent the rates in miles per hour and the times in hours. Have the computer print the chart below.

```
VARYING THE RATE AND TIME
RATES     TIMES     DISTANCES
(MPH)     (HR.)     (MI.)
15        3         45
22        7         154
25        5         125
30        6         180
34        4         136
39        2         78
```

DEVELOPMENT:

1. Ask the students to use the flowchart developed in the last lesson and the planned output included in the statement of the problem to code a program as follows.

Program

```
10 PRINT "VARYING THE RATE AND TIME"
20 PRINT "RATES", "TIMES", "DISTANCES"
30 PRINT "(MPH)", "(HR.)", "(MI.)"
40 READ R,T
50 IF R = -99 THEN 100
```

```
 60      D = R*T
 70      PRINT R,T,D
 80      GOTO 40
 90      DATA 15,3,22,7,25,5,30,6,34,4,39,2,-99,-99
100      END
```

2. If printers are available, ask each student to hand in a lisitng of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.

3. Circulate around the laboratory assisting students as needed.

4. Group the students who are unable to write the program. Work through some of the coding with the students.

5. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

1. a) Trace each program.
   b) Verify that your trace is correct by entering and running the program.
   c) Edit the program so that the OUT OF DATA message will be eliminated.

i)

```
10 READ R
20 N = R+1
30 PRINT"THE NUMBER AFTER";R;"IS";N
40 DATA 5,3,8
50 GOTO 10
60 END
```

ii)

```
10 PRINT "NUMBER","DOUBLE","TRIPLE"
20 READ N
30 D = 2*N
40 T = 3*N
50 PRINT N,D,T
60 DATA 1,4,7,5,2
70 GOTO 20
80 END
```

For the problem that follows, complete the instructions.

a) Write an analysis which includes the development of an algorithm.

b) Write a plan for a computer solution to the problem.

c) Develop a flowchart.

d) Design a planned output.

e) Code a program and RUN it on the computer.

2. Problem: Have the computer read five pairs of values to represent the distances traveled in miles and the times in hours. Have it then print a chart showing the distances (in miles), times (in hours), and the rates (in miles per hour). Use the following data:440,8,192,4,325, 6.5,636,12,327.6,6. (Be sure there is no OUT OF DATA message.)

SUMMARY:

What is the purpose of a "flag" when used in conjunction with a READ-DATA statement?

HOMEWORK:

1. Use the formula, D = RT, to find the missing value in each of the following.

a)	R = 48 mph T = 6 hr. D = ?	b)	R = 43 mph T = ? D = 387 mi.
c)	R = ? T = 3.5 hr. D = 192.5 mi.	d)	R = ? T = 2 hr. 12 min. D = 118.8 mi.

2. Mr. Suarez left his home in his car at 8:00 a.m. and drove to a city 308 kilometers away. He arrived there at 3:45 p.m. He had stopped $2\frac{1}{4}$ hours on the road for lunch and rest.
   a) How many hours did he spend driving his car?
   b) What was the average rate at which he drove, expressed in kilometers per hour?

3. Mr. and Mrs. Jones took a six-day car trip. When they left home, the odometer on their car read 19,972 miles. When they returned home, it read 21,112 miles.
   a) How many miles did they travel on the trip?
   b) How many miles did they average per day?
   c) At an average cost of 22.5 cents per mile, what was the total cost of transportation?
   d) If they averaged 38 miles per gallon of gasoline, how much did the gasoline for the trip cost at $1.19 a gallon?

4. For the problem that follows, execute the instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan for a computer solution to the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

Problem: Have the computer read five pairs of values to represent the distances traveled in miles and the rates in miles per hour. Have it then print a chart showing the distances (in miles), the rates (in miles per hour), and the times (in hours). Use the following data: 416,52, 147,49,279,45,321,53.5,196.8,49.2. (Be sure there is no OUT OF DATA message.)

HOMEWORK SOLUTIONS:

1. a) D = 228 mi. b) T = 9 hr.
   c) R = 55 mph d) R = 54 mph

2. a) $5\frac{1}{2}$ hr. or 5.5 hr. b) 56 km per hr.

3. a) 1140 mi. c) $256.50
   b) 190 mi. d) $35.70

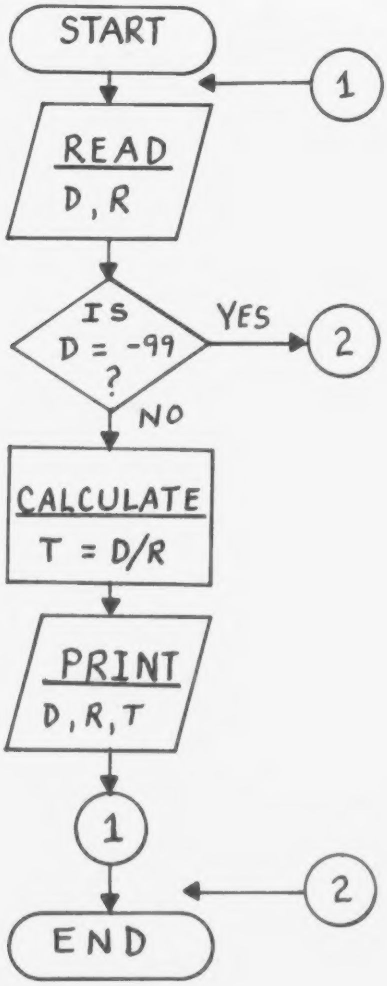
4a) Analysis

T = D/R where memory cell D stores the distance,
memory cell R stores the rate, and
memory cell T stores the time.

4b) Plan

```
1)  Read values for the distance and the rate:  D,R.
2)  Check:  Is D = -99?
      a) If yes, then go to 7.
      b) If no, continue.
3)  Calculate the time:  T = D/R.
4)  Print D,R,T.
5)  Go to 1.
6)  Data 416,52,147,49,279,45,321,53.5,196.8,49.2,-99,-99
7)  End
```

4c) Flowchart


START
1
READ
D, R
IS
D = -99
?
YES
2
NO
CALCULATE
T = D/R
PRINT
D, R, T
1
2
END

4d) Planned Output

```
VARYING THE DISTANCE AND RATE
DISTANCES      RATES        TIMES
(IN MI.)       (IN MPH)     (IN HR.)
416            52           8
147            49           3
279            45           6.2
321            53.5         6
196.8          49.2         4
```

4e) Program

```
10  PRINT "VARYING THE DISTANCE AND RATE"
20  PRINT "DISTANCES","RATES","TIMES"
30  PRINT "(IN MI.)","(IN MPH)","(IN HR.)"
40  READ D,R
50  IF D = -99 THEN GOTO 100
60  T = D/R
70  PRINT D,R,T
80  GOTO 40
90  DATA 416,52,147,49,279,45,321,53.5,196.8,49.2,-99,-99
100 END
```

Lesson: 57

AIM: How do we determine the area of a trapezoid?

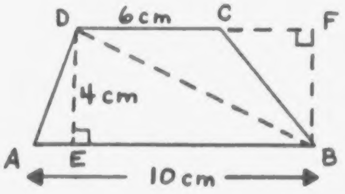
PERFORMANCE OBJECTIVES: The student will be able to...

1. identify the bases and height (altitude) of a trapezoid;
2. demonstrate his/her understanding of determining the area of a trapezoid by developing a plan (flowchart) for a computer solution;
3. convert from one metric unit of length to another.

VOCABULARY: bases, height, trapezoid, diagonal, parallel lines, equidistant, perpendicular, factors, addends, distributive property of multiplication over addition

Symbols: ⊥, ||, ∟

MOTIVATION:



Given: Trapezoid ABCD, with AB DC and diagonal $\overline{BD}$ drawn. In triangle ABD, $\overline{DE}$ $\overline{AB}$ and in triangle BCD, $\overline{BF}$ $\overline{DC}$ extended. $\overline{DE}$ = 4 cm, $\overline{AB}$ = 10 cm and $\overline{DC}$ = 6 cm

a) What is the area of triangle ABD?
b) What is $\overline{BF}$?
c) What is the area of triangle BCD?
d) What is the area of trapezoid ABCD?

DEVELOPMENT:

1a. Use colored chalk to draw the diagram (in the motivation) on the chalkboard. Use different colors for each triangular region. A model drawn on construction paper or oaktag can also be used in moving the triangles around to illustrate that △DCB is obtuse; therefore, its height is in the exterior and has the same length as that of △ABD.

1b. You might wish to review some of the vocabulary and symbols listed in the "Given" with the students initially; for e.g., "diagonal", "trapezoid", "$\overline{DE}$" means the length of $\overline{DE}$, "$\overline{DE} \perp \overline{AB}$" means line segment $\overline{DE}$ is perpendicular to line segment $\overline{AB}$, " ∟ " indicates a right angle is formed, "$\overline{AB} \parallel \overline{DC}$" means line segment $\overline{AB}$ is parallel to line segment $\overline{DC}$, ...

2. Review with the students the formula for determining the area of a triangle. Ask the students to identify the base and the height of △ABD. Ask them to substitute into the formula as follows:

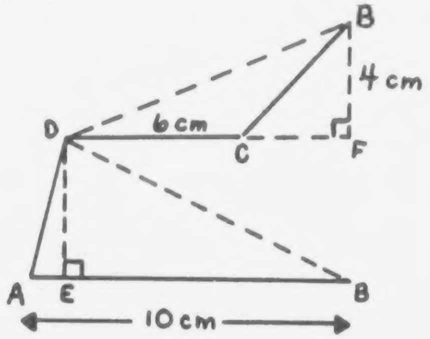
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{5}{\cancel{10}}}{1} \times \frac{4}{1}$$

$$A = \frac{20}{1} = 20 \text{ sq. cm}$$

3. Ask, "If a trapezoid is a quadrilateral with exactly one pair of parallel sides, which sides of trapezoid ABCD are parallel? What is the length of $\overline{BF}$?" Point out to the students that parallel lines are equidistant; e.g., always the same length apart. Use the construction paper (oaktag) model or cut a strip of paper equal in length to that of $\overline{DE}$ and compare it with the length of $\overline{BF}$. Ask, "If $\overline{AB}$ was not parallel to $\overline{CD}$, would $\overline{DE}$ and $\overline{BF}$ have the same length?" Illustrate this on the chalkboard.

4. Ask the students to recognize that $\overline{BF}$ is the height to the base $\overline{DC}$ extended in △BCD. If you have a model, raise it as follows:



You can also turn the trapezoid around. In either case, ask the students to use the formula for the area of a triangle as follows:

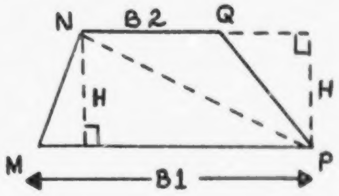
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{3}{\cancel{6}}}{1} \times \frac{4}{1}$$

$$A = \frac{12}{1} = 12 \text{ sq. cm}$$

5. Then ask the students to recognize that the area of trapezoid ABCD is the sum of the areas of △ABD and △BCD. Ask, "What is the area of trapezoid ABCD?"

6. Say to the students that we will generalize what we just did in order to develop a formula for the area of a trapezoid.



   a) Draw a trapezoid on the chalkboard as shown and add the symbols and line segments one at a time. Ask, "Which sides appear to be parallel?" Say to the students that these are the bases of the trapezoid whose lengths will be represented by B1 and B2. Label the diagram.

   b) Say to the students that we will draw a line segment from N perpendicular to $\overline{MP}$. The length of this line segment will be represented by H. Say to the students that the length of this segment represents the height of trapezoid MPQN. Label the diagram.

   c) Draw diagonal $\overline{NP}$. Ask the students to notice that two triangular regions, △MPN and △NQP, are formed. Extend $\overline{NQ}$ and draw a line segment from P perpendicular to $\overline{NQ}$ extended. Ask, "Why can we also use H to represent the length of this line segment?"

d) Say to the students that the area of $\triangle$MPN is represented by $\frac{1}{2}\cdot B1\cdot H$ and the area of $\triangle$NQP is represented by $\frac{1}{2}\cdot B2\cdot H$. Ask, "How can we represent the area of the trapezoid?" Write the response on the chalkboard as follows:

Area of the trapezoid = Area of $\triangle$MPN + Area of $\triangle$NQP
Area of the trapezoid = $\frac{1}{2}\cdot B1\cdot H$ + $\frac{1}{2}\cdot B2\cdot H$

Ask, "Which factors are common to both addends?" Point out to the students that by using the distributive property of multiplication over addition, we can write this as: Area of the trapezoid = $\frac{1}{2}H(B1 + B2)$ or

$$A = \frac{1}{2}H(B1 + B2).$$

Stress that the measurements for the height and bases must be expressed in the same unit of measure. Ask the students to redo the motivation problem using the formula just derived. Then ask them to complete Applications 1-3.

7. Present the problem that follows for a computer solution.

<u>Problem</u>: Have the computer read the dimensions (expressed in cm) of four trapezoids. Then have it print the chart below showing the height, bases, and area of each triangle.

```
FINDING THE AREAS OF TRAPEZOIDS
HEIGHTS      BASES       AREAS
(IN CM)      (IN CM)     (IN SQ. CM)
11           14,10       132
12           20,16       216
14           16,12       196
9            12,5        76.5
```

8. Elicit the analysis below from the students.

<u>Analysis</u>

a) Ask, "What algorithm will we have the computer use to calculate the area of each trapezoid?" Elicit $A = \frac{1}{2}H(B1+B2)$ where A represents the area,
H represents the height, and
B1 and B2 represent the bases.

b) Ask, "Since we will be reading the data, how can we avoid an OUT OF DATA message?" Elicit that we must include a "flag" to signal the computer that there is no more data.

9. Develop the plan below with the students.

Plan

1) Read the dimensions (height, base1, base2) of each trapezoid: H, B1, B2.
2) Check: Is H = -1?
    a) If yes, then go to 8.
    b) If no, continue.
3) Calculate the area of the trapezoid: $A = \frac{1}{2}H(B1+B2)$.
4) Print H,B1,B2,A.
5) Go to 1.
6) Data 11,14,10,12,20,16,14,16,12,9,12,5
    Ask the students to notice that the data is written as height, base1, base2, height, base1, base2,... which corresponds to the way it is read.
7) Data -1,-1,-1
    Ask the students to notice that three additional numbers are added to the data. Remind them that we can use any numbers here, but it is a good idea to make them distinguishable from the rest of the data in the program.
8) End.

b) Ask the following questions as you develop the plan.
    i) What values do we want to read into the computer? Say that we will store the measures for the height and bases in memory cells H, B1, and B2.
    ii) Each time a triple of numbers is read, what check do we want to make on the value in memory cell H in order to signal (flag) the computer that there is no more data?

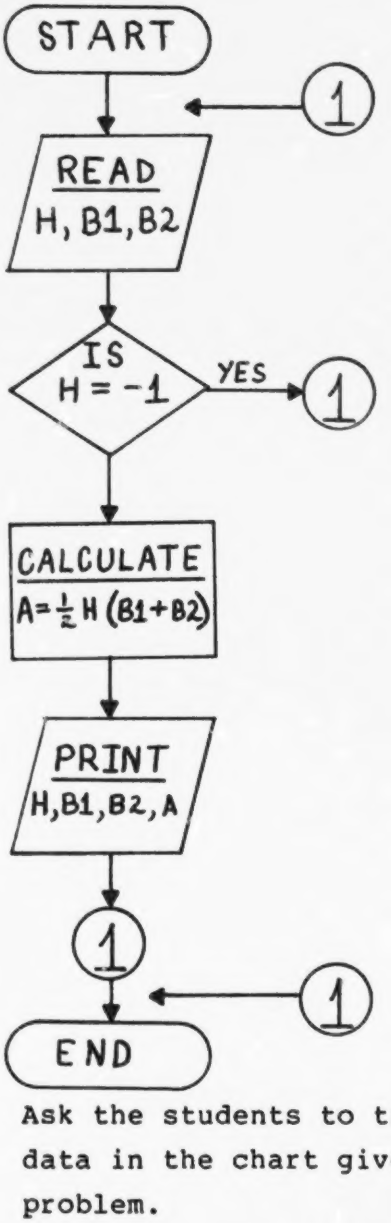
iii) If the value in memory cell H is -1, what do we want the computer to do?

iv) If the value in memory cell H is not -1, what algorithm do we want the computer to use in order to calculate each area?

v) What do we want the computer to display?

vi) In what order do we list the data in the DATA statement?

10. a) Ask the students to develop a flowchart as follows.


START
1
READ
H, B1, B2
IS
H = -1
YES
1
CALCULATE
A = ½ H (B1 + B2)
PRINT
H, B1, B2, A
1
1
END

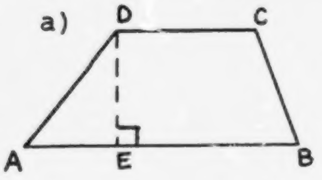
b) Ask the students to trace the program using the data in the chart given in the statement of the problem.

c) Ask, "If we are only checking memory cell H, why do we need to put three extra numbers in a DATA statement?" Discuss with the students that when the computer goes back to read a fifth triple of numbers, memory cells H, B1 and B2 must all be read or an OUT OF DATA message will be displayed.

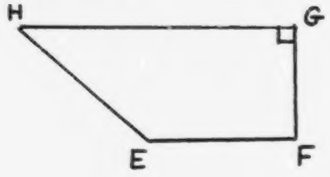
APPLICATIONS:

1. Name the bases and height in each trapezoid shown below.

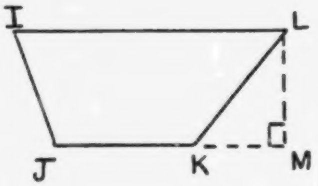
a)



b)

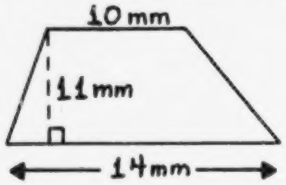


c)

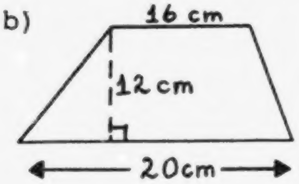


2. Compute the areas of the trapezoids pictured below.

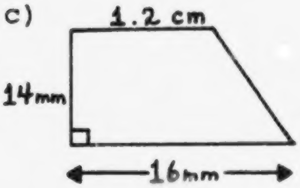
a)



b)



c)



3. Compute the areas of the trapezoids with the following dimensions.
   - a) H= 5 in., B1 = 10 in., B2 = 6 in.
   - b) H = 10 ft., B1 = 20 ft., B2 = 16 ft.
   - c) H = 9 cm, B1 = 12 cm, B2 = 5 cm
   - d) H = 18 mm, B1 = 3 cm, B2 = 20 mm

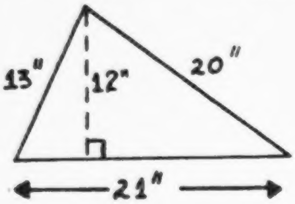
SUMMARY:

1. The measurements of which dimensions do we have to be given in order to determine the area of a trapezoid?
2. How do we find the area of a trapezoid?

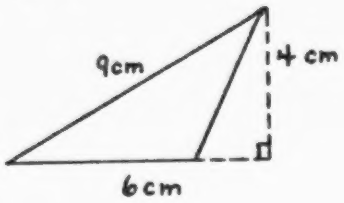
HOMEWORK:

1. Find the area of each polygon below.

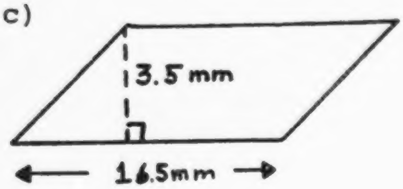
a)



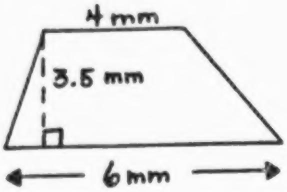
b)



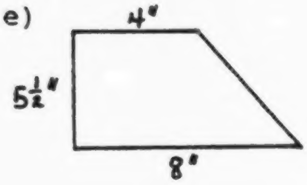
c)



d)



e)



2. Compute the area of the trapezoids with the following dimensions.
   a) H = $2\frac{1}{2}$ in., B1 = 8 in., B2 = 4 in.
   b) H = 3.2 m, B1 = 14.6 m, B2 = 9.3 m
   c) H = 4 ft., B1 = $12\frac{1}{2}$ ft., B2 = $8\frac{1}{2}$ ft.
   d) H = 3.7 cm, B1 = 42 mm, B2 = 27 mm

HOMEWORK SOLUTIONS:

1. a) 126 sq. in. d) 17.5 sq. mm
   b) 12 sq. cm e) 32.5 sq. in.
   c) 57.75 sq. mm

2. a) 15 sq. in. c) 42 sq. ft.
   b) 38.24 sq. m d) 12.765 sq. cm or 127.65 sq. mm

Lesson: 58L

AIM: How do we code a program to display the areas of trapezoids when given the measurements of the heights and the bases?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to have the computer find the areas of a trapezoids, given the measures of the heights and the bases;
2. enter, run, and correct his/her program;
3. incorporate a "flag" to elimlinate the OUT OF DATA message in a program that involves the use of a READ-DATA statement;
4. convert from one metric unit of length to another.

VOCABULARY: trapezoid, base, height, altitude

MOTIVATION:

Code a program that will solve the problem below.

Problem: Have the computer read the dimensions (expressed in centimeters) of four trapezoids. Then have it print the chart below showing the height, bases, and area of each trapezoid.

```
FINDING THE AREAS OF TRAPEZOIDS
HEIGHTS     BASES      AREAS
(IN CM)     (IN CM)    (IN SQ. CM)
11          14,10      132
12          20,16      216
14          16,12      196
9           12,5       76.5
```

DEVELOPMENT:

1. Ask the students to use the flowchart developed in the last lesson and the planned output included in the statement of the problem to code a program as follows:

Program

```
10 PRINT "FINDING THE AREAS OF TRAPEZOIDS"
20 PRINT "HEIGHTS", "BASES", "AREAS"
```

```
30  PRINT "(IN CM)", "(IN CM)","(IN SQ. CM)"
40  READ H,B1,B2
50  IF H = -1 THEN 110
60  A = 1/2*H*(B1 + B2)
70  PRINT H,B1;",";B2,A
80  GOTO 40
90  DATA 11,14,10,12,20,16,14,16,12,9,12,5
100 DATA -1,-1,-1
110 END
```

2. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.
3. Circulate around the laboratory assisting students as needed.
4. Group the students who are unable to write the program. Work through some of the coding with the students.
5. Ask the students who complete the problem presented, to solve the problems described in Applications.

APPLICATIONS:

1. Fill in the table below by finding the area of each trapezoid, expressed in square centimeters.

HEIGHTS	BASES	AREAS
30 mm	5 cm, 7 cm	? sq. cm
10 mm	6 cm, 4 cm	? sq. cm
25 mm	4 cm, 8 cm	? sq. cm
24 mm	7 cm, 3 cm	? sq. cm
8 mm	2 cm, 8 cm	? sq. cm

2. For the problem that follows, complete the instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan for a computer solution to the problem.
   c) Develop a flowchart.
   d) Design a planned output.

e) Code a program and RUN it on the computer.

Problem: Have the computer read a value for the measure of the height (expressed in mm) and values for the measures of the bases (expressed in cm) for each of five trapezoids. Then have it print a chart showing the height (expressed in mm), the bases (expressed in cm) and the area (expressed in sq. cm) of each trpezoid. Use the following data (obtained from Application 1): 30,5,7, 10,6,4,25,4,8,24,7,3,8,2,8. (Be sure that an OUT OF DATA message does not appear.)

SUMMARY:

How was the "flag" used to eliminate the OUT OF DATA message in the program in today's lesson?

HOMEWORK:

1. Fill in the table below by finding the area of each trapezoid, expressed in square millimeters.

HEIGHTS	BASES	AREAS
30 mm	5 cm, 7 cm	? sq. mm
10 mm	6 cm, 4 cm	? sq. mm
25 mm	4 cm, 8 cm	? sq. mm
24 mm	7 cm, 3 cm	? sq. mm
8 mm	2 cm, 8 cm	? sq. mm

2. For the problem that follows, complete the instructions.
   a) Write an analysis which includes the development of an algorithm.
   b) Write a plan for a computer solution to the problem.
   c) Develop a flowchart.
   d) Design a planned output.
   e) Code a program.

Problem: Have the computer read a value for the measure of the height (expressed in mm) and values for the measures of the bases (expressed in cm) for each of five trapezoids. Then have it print a chart showing the height (expressed in mm), the bases (expressed in cm), and the area (expressed in sq. mm) of each trapezoid. Use the following data (obtained from Problem 1): 30, 5, 7, 10, 6, 4, 25, 4, 8, 24, 7, 3, 8,2, 8. (Be sure that an OUT OF DATA message does not appear.)

(Note to the teacher: Ask the students to compare the areas expressed in sq. cm (from the Applications) and the areas expressed in sq. mm (from the Homework).) Ask, "How many times the area expressed in sq. cm is the area expressed in sq. mm?"

HOMEWORK SOLUTIONS:

1. AREAS
   1800 sq. mm
   500 sq. mm
   1500 sq. mm
   1200 sq. mm
   400 sq. mm

2a) Analysis

M1 = 10*B1 where memory cell B1 stores the measure of base 1 expressed in cm and memory cell M1 stores the measure of base 1 expressed in mm.

M2 = 10*B2 where memory cell B2 stores the measure of base 2 expressed in cm and memory cell M2 stores the measure of base 2 expressed in mm.

$A = \frac{1}{2} H(M1+M2)$ where memory cells M1 and M2 store the measures of the dimensions, both expressed in mm, and A stores the area.

2b) Plan

```
1)  Read values for the dimensions (height, base 1,
    base 2) of each trapezoid:  H,B1,B2.
2)  Check:  Is H = -1?
      a) If yes, then go to 10.
      b) If no, continue.
3)  Convert the length of base 1 to mm:  M1 = 10*B1.
4)  Convert the length of base 2 to mm:  M2 = 10*B2.
```

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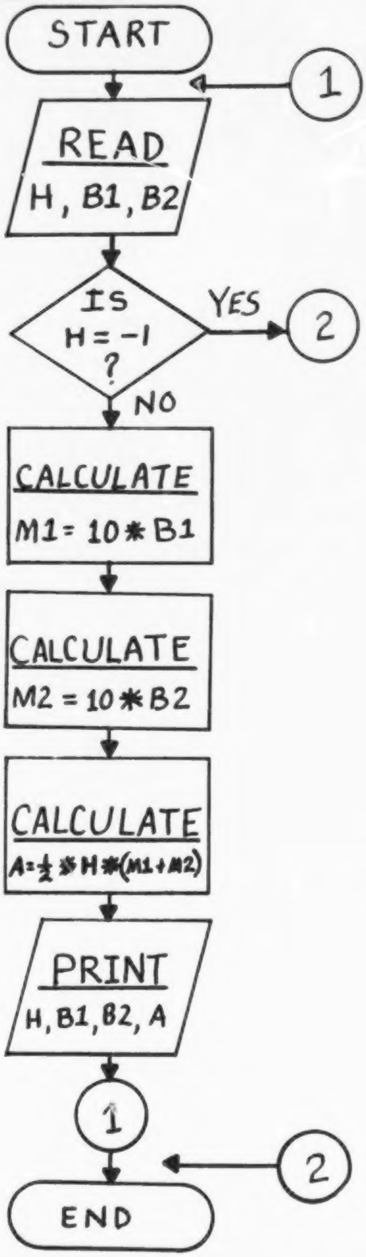
5) Calculate the area of the trapezoid:
$A = \frac{1}{2}H(M1+M2)$.
6) Print H, B1, B2, A.
7) Go to 1.
8) Data 30,5,7,10,6,4,25,4,8,24,7,3,8,2,8
9) Data -1,-1,-1
10) End.

2c) Flowchart


START
1
READ
H, B1, B2
IS
H = -1
?
YES
2
NO
CALCULATE
M1 = 10 * B1
CALCULATE
M2 = 10 * B2
CALCULATE
A = ½ * H * (M1 + M2)
PRINT
H, B1, B2, A
1
2
END

2d) Planned Output

```
FINDING THE AREAS OF TRAPEZOIDS
HEIGHTS      BASES      AREAS
(IN MM)      (IN CM)    (IN SQ. MM)
3            5,7        1800
10           6,4        500
25           4,8        1500
24           7,3        1200
8            2,8        400
```

2e) Program

```
10  PRINT "FINDING THE AREAS OF TRAPEZOIDS"
20  PRINT "HEIGHTS","BASES","AREAS"
30  PRINT "(IN MM)","(IN CM)","(IN SQ. MM)"
40  READ H,B1,B2
50  IF H = -1 THEN GOTO  130
60  M1 = 10*B1
70  M2 = 10*B2
80  A = 1/2*H*(M1+M2)
90  PRINT H,B1;",";B2,A
100 GOTO 40
110 DATA 3,5,7,10,6,4,25,4,8,24,7,3,8,2,8
120 DATA -1,-1,-1
130 END
```

Lesson: 59

AIM: How do the perimeters of rectangles with the same area compare?

PERFORMANCE OBJECTIVES: The student will be able to...

1. find the area of a rectangle;
2. find the perimeter of a rectangle;
3. find all integers that can be used as measures for the dimensions of rectangles with a given area;
4. demonstrate his/her understanding of the above concepts by incorporating them into a plan (flowchart) for a computer solution to the problem.

VOCABULARY: length, width, rectangle, dimensions, factors, integer (integral), square, minimum, natural number

MOTIVATION:

Name two integers that can be used as the measures of the dimensions of a rectangle whose area is 36 sq. cm.

DEVELOPMENT:

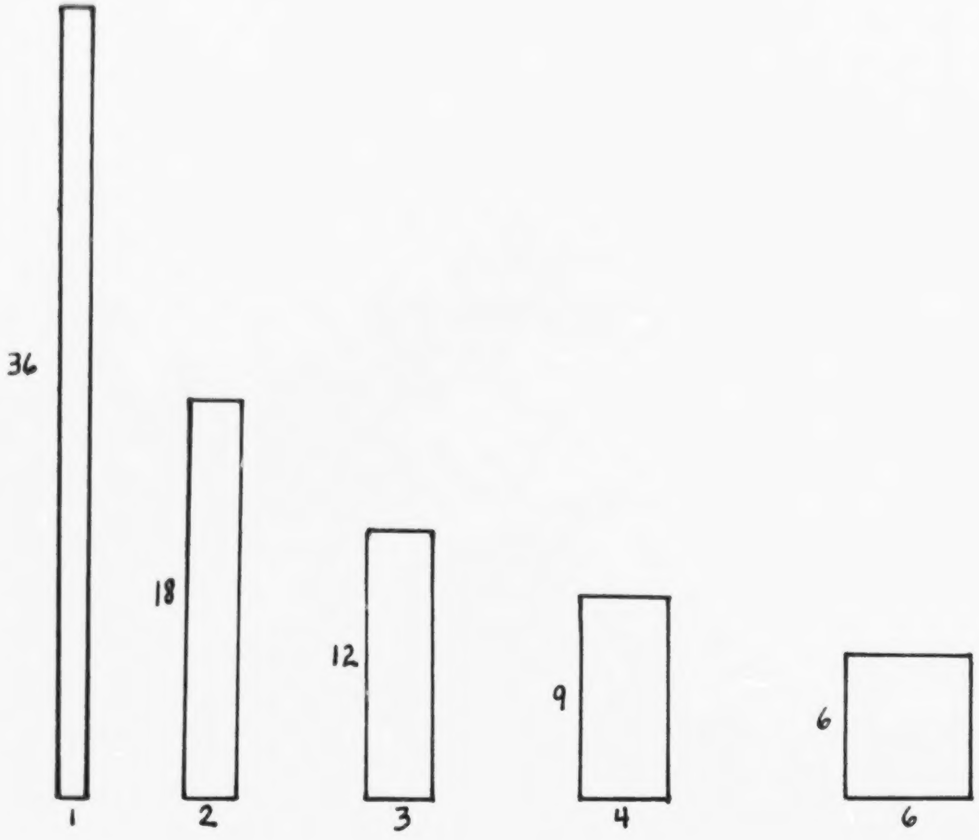
1. Ask, "What is the formula for the area of a rectangle which we discussed earlier this term?"
   Elicit $A = lw$ where A represents the area,
   l represents the length, and
   w represents the width.

   Ask the students to use the formula as follows:
   $$A = lw$$
   $$36 = lw$$
   Say to the students that in order to solve the problem (presented in the motivation), we must find two numbers whose product is 36. Elicit some possibilities from the students. Ask, "If we wanted all the possible positive integers whose products are 36, how would we determine them?" Ask the students to determine all the factors of

36 in a systematic way;
e.g.,

LENGTHS (IN CM)	WIDTHS (IN CM)
36	1
18	2
12	3
9	4
6	6

2. Ask, "What is the perimeter of each rectangle?" You may wish to ask the students to draw on graph paper the rectangles whose dimensions are listed above. This will clarify that when finding area, they are finding the measure of the interior; while, when finding perimeter, they are finding the measure of the boundary;
e.g.,



Ask, "Which rectangle appears to have the smallest perimeter? What name is given to a rectangle whose dimensions are equal in length? In order to verify this, rather than counting the sides of the squares (illustrate this on the chalkboard or on an overhead projector), what formula can we use to determine the perimeter of each rectangle?" Ask the students to use one of the formulas for each rectangle;

e.g.,

$P = 2l + 2w$	$P = 2(l + w)$
$P = 2 \cdot 36 + 2 \cdot 1$	$P = 2(36 + 1)$
$P = 72 + 2$	$P = 2(37)$
$P = 74$ cm	$P = 74$ cm

Ask, "What mathematical principle is used to show that

$2l + 2w = 2(l + w)$?"

Ask the students to add another column to the chart showing lengths and widths and have them list the perimeters;

e.g.,

LENGTHS (IN CM)	WIDTHS (IN CM)	PERIMETERS (IN CM)
36	1	74
18	2	40
12	3	30
9	4	26
6	6	24

Ask the students to notice that as predicted, the square has the smallest perimeter.

3. Ask the students to complete the two problems in the Applications. After going over the Applications, ask the students to generalize that the square has the minimum perimeter for all rectangles with a given fixed area.

4. Present the problem below for a computer solution.

Problem: Have the computer display the following chart where the lengths and widths are entered and the computer calculates the perimeters.

```
COMPARING THE PERIMETERS OF RECTANGLES
LENGTHS    WIDTHS    PERIMETERS
(IN CM)    (IN CM)   (IN CM)
36         1         74
18         2         40
12         3         30
9          4         26
6          6         24
```

5. Elicit the analysis that follows from the students.

   Analysis

   a) Ask the students to notice that there are no question marks in the chart. Ask, "How can we enter the lengths and widths without question marks appearing in the output?" Point out to the students that an OUT OF DATA message does not appear. Ask, "How can we avoid the OUT OF DATA message?"

   b) Ask, "What algorithm do we have the computer use in order to determine the perimeter of each rectangle?"

6. Develop the plan that follows with the students.

   a) Plan

      1) Read values for the length and width of each rectangle: L, W.
      2) Check: Is L = 0?
         a) If yes, then go to 8.
         b) If no, continue.
      3) Calculate the perimeter: P = 2L + 2W.
      4) Print L, W, P.
      5) Go to 1.
      6) Data 36,1,18,2,12,3,9,4,6,6

         Ask the students to notice that the data is written as length, width, length, width,... This is because the computer will read the values in that order.
      7) Data 0,0

         Ask the students to notice that two additional numbers are added to the data. Remind them that we can use any numbers here but that it is advisable to make them distinguishable from the rest of the data that will be used in the program.

8) End.

b) Ask the following questions as you develop the plan.

   i) What values do we want to read into the computer? Say that we will store the length and width in memory cells L and W.

   ii) Each time a pair of numbers is read, what check do we want to make on the value in memory cell L in order to signal (flag) the computer that there is no more data?

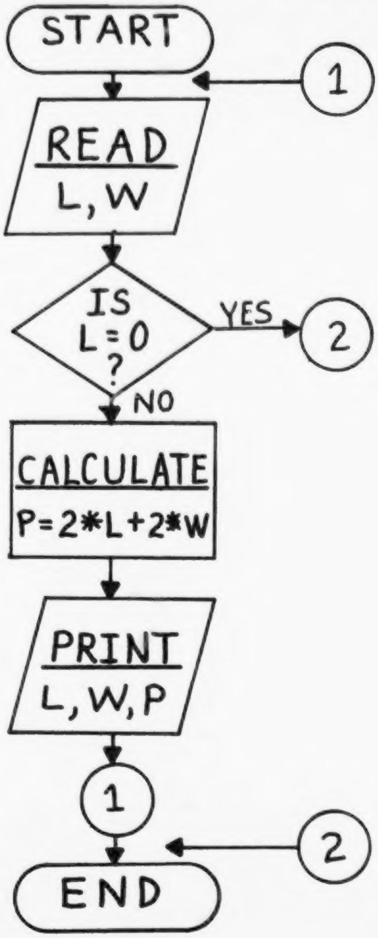
   iii) If the value in memory cell L is 0, what do we want the computer to do?

   iv) If the value in memory cell L is not 0, what algorithm do we want the computer to use in order to calculate the perimeter?

   v) What do we want the computer to display?

   vi) In what order do we list the data in the DATA statement?

9. a) Ask the students to develop a flowchart as follows:



b) Ask the students to trace the flowchart using the data in the chart given in the statement of the problem.

c) Ask, "If we are only checking memory cell L, why do we need to put two extra numbers in the DATA statement?" Discuss with the students that when the computer goes back to read a sixth pair of numbers, memory cells L and W must both be read or an OUT OF DATA message will be displayed.

APPLICATIONS:

1. a) Find all pairs of integers that can be used as the measures of the dimensions of a rectangle whose area is 64 sq. cm.
   b) Find the perimeter of each rectangle whose dimensions have been determined in 1a.
   c) Which rectangle has the smallest perimeter?
2. a) Find all pairs of integers that can be used as the measures of the dimensions of a rectangle whose area is 144 sq. cm.
   b) Find the perimeter of each rectangle whose dimensions have been determined in 2a.
   c) Which rectangle has the smallest perimeter?

SUMMARY:

1. How do we determine all positive integral factors of a given natural number?
2. How do we find the perimeter of a rectangle?
3. If the area is fixed (always the same number), which rectangle has the minimum perimeter?

HOMEWORK:

1. Find the area and perimeter of a rectangle whose dimensions are:

a) 24 cm by 7 cm c) $12\frac{5}{8}$" by 48"
b) 1.5 yd. by 8.3 yd.

2. How many tiles, each of whose area is 1 sq. ft., are needed to cover the floor of a rectangular hall, 3 yd. wide and 20 yd. long?

3. A rectangular garden plot is 12 meters wide. If it contains 288 square meters, how long is it?

4. How much larger is the area of a rectangle that is 10 cm by 25 cm than that of a rectangle that is 12 cm by 20 cm?

5. A rectangular lot is 100 feet long and 55 feet wide.
   a) How many feet of fencing are required to enclose the lot?
   b) At $6.50 per foot, how much will the fencing cost?

HOMEWORK SOLUTIONS:

1. a) A = 168 cm^2, p = 62 cm
   b) A = 12.45 $yd.^2$, p = 19.6 yd.
   c) A = 606 sq. in., p = 121.25 in.

2. 540 tiles

3. 24 meters long

4. The area of the rectangle that is 10 cm by 25 cm, 250 cm^2, is 10 cm^2 larger than the area of the rectangle that is 12 cm by 20 cm.

5. a) 310 feet b) $2015

Lesson: 60L

AIM: How do we code a program to have the computer determine the perimeters of rectangles with a fixed area?

PERFORMANCE OBJECTIVES: The student will be able to...

1. code a program to have the computer find the perimeters of all rectangles with a fixed area;
2. enter, run, and correct his/her program;
3. incorporate a "flag " to avoid the OUT OF DATA message in a program that involves the use of a READ-DATA statement;
4. find integers that can be used as measures for the dimensions of rectangles with a given perimeter.

VOCABULARY: rectangle, length, width, dimensions, integer (integral), factors, minimum, maximum, square

MOTIVATION:

Code a program that will solve the problem below.

Problem: Have the computer display the chart below, where the lengths and widths are entered and the computer calculates the perimeters.

```
COMPARING THE PERIMETERS OF RECTANGLES
LENGTHS         WIDTHS          PERIMETERS
(IN CM)         (IN CM)         (IN CM)
36              1               74
18              2               40
12              3               30
9               4               26
6               6               24
```

DEVELOPMENT:

1. Ask the students to use the flowchart developed in the last lesson and the planned output included in the statement of the problem to code a program as follows:

Program

```
10  PRINT "COMPARING THE PERIMETERS OF RECTANGLES"
20  PRINT "LENGTHS", "WIDTHS", "PERIMETERS"
30  PRINT "(IN CM)", "(IN CM)", "(IN CM)"
40  READ L,W
50  IF L = 0, THEN 110
60  P = 2*L + 2*W
70  PRINT L,W,P
80  GOTO 40
90  DATA 36,1,18,2,12,3,9,4,6,6
100 DATA 0,0
110 END
```

2. If printers are available, ask each student to hand in a listing of the program after he/she has made all corrections. If printers are not available, ask each student to run his/her program for you to grade.
3. Circulate around the laboratory assisting as needed.
4. Group the students who are unable to write the program. Work through some of the coding with these students.
5. Ask the students who complete the problem presented, to solve the problems described in the Applications.

APPLICATIONS:

1. a) Find all pairs of integers that can be used as the measures of the dimensions of a rectangle whose perimeter is 24 cm.
   b) Prepare a table listing the lengths (in cm), the widths (in cm), and the areas (in sq. cm) of all the rectangles whose dimensions were determined in 1a.
   c) Which rectangle has the largest area?
2. a) Find six pairs of distinct integers that can be used as the measures of the dimensions of a rectangle whose perimeter is 100 cm.
   b) Prepare a table listing the lengths (in cm), the widths (in cm), and the areas (in sq. cm) of all the rectangles whose dimensions were determined in 2a.

c) What is the length of a side of a square whose perimeter is 100 cm?

d) What is the area of a square whose perimeter is 100 cm?

e) How does the area of the square compare with the areas of the other rectangles (in the table) whose perimeter is 100 cm?

3. For the problem below, complete the following instructions.

    a) Write an analysis of the problem that leads to an algorithm.

    b) Write a plan that outlines how you use the algorithm to solve the problem.

    c) Develop a flowchart.

    d) Design a planned output.

    e) Code a program and RUN it on the computer.

Problem: Have the computer read pairs of measures representing the lengths and widths of six rectangles whose perimeter is 144 cm. Then have it print a chart showing the lengths (in cm), the widths (in cm), and the areas (in sq. cm). Use the following data: 60,12,47, 25,45,27,42,30,40,32,36,36. (Be sure that an OUT OF DATA message does not appear.)

SUMMARY:

1. How do we find the area of a rectangle?
2. If the perimeter is fixed, which rectangle has the maximum area?

HOMEWORK:

Complete the instructions for the problem that follows.

a) Write an analysis of the problem that leads to an algorithm.

b) Write a plan that outlines how you use the algorithm to solve the problem.

c) Develop a flowchart.

d) Design a planned output.

e) Code a program.

<u>Problem</u>: Have the computer read pairs of measures representing the lengths and widths of five rectangles. Then have it print a chart showing the lengths (in mm), the widths (in mm), and the areas (in sq. mm). Use the following data: 40,3,68,10,8.7,5,6,3.2,9.7,4.1. (Be sure that an OUT OF DATA message does not appear.)

HOMEWORK SOLUTIONS:

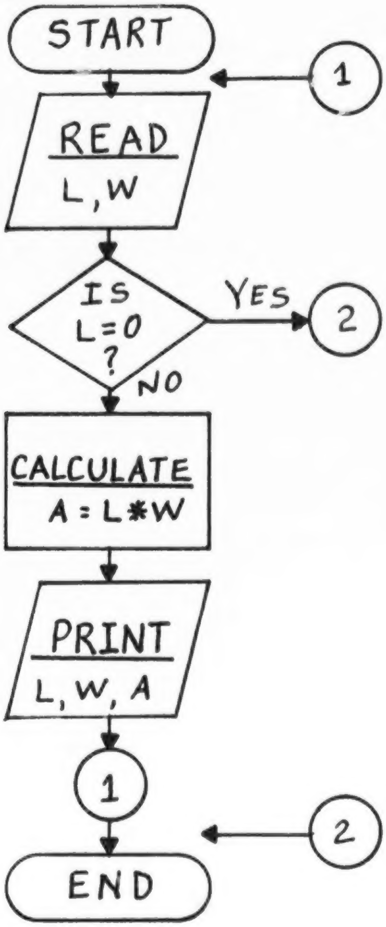
a) <u>Analysis</u>

A = LW where memory cell L stores the measure of the length,
memory cell W stores the measure of the width and
memory cell A stores the measure of the area.

b) <u>Plan</u>

```
1)  Read values for the length and width of each
    rectangle:  L,W.
2)  Check:  Is L = 0?
      a) If yes, then go to 8.
      b) If no, continue.
3)  Calculate the area:  A = LW.
4)  Print L,W,A.
5)  Go to 1.
6)  Data 40,3,68,10,8.7,5,6,3.2,9.7,4.1
7)  Data 0,0
8)  End
```

c) Flowchart


START
1
READ
L, W
IS
L=0
?
YES
2
NO
CALCULATE
A = L*W
PRINT
L, W, A
1
2
END

d) Planned Output

```
FINDING THE AREAS OF RECTANGLES
LENGTHS      WIDTHS       AREAS
(IN MM)      (IN MM)      (IN SQ. MM)
40           3            120
68           10           680
8.7          5            43.5
6            3.2          19.2
9.7          4.1          39.77
```

e) Program

```
10  PRINT "FINDING THE AREAS OF RECTANGLES"
20  PRINT "LENGHTS","WIDTHS","AREAS"
30  PRINT "(IN MM)","(IN MM)","(IN SQ. MM)"
40  READ L,W
50  IF L=0 THEN GOTO  110
60  A = L*W
70  PRINT L,W,A
80  GOTO 40
90  DATA 40,3,68,10,8.7,5,6,3.2,9.7,4.1
100 DATA 0,0
110 END
```

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